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Study on disturbance attenuation of cellular neural networks with time-varying delays

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ABSTRACT

The paper is concerned with the problem of disturbance attenuating controller design for delayed cellular neural networks (DCNNs). Via combining four different states cases in DCNNs and applying Razumikhin function analysis, a feedback control law in the form of linear matrix inequality (LMI) is derived for guaranteeing disturbance attenuation of the closed systems. Finally, a numerical example of DCNNs is given to indicate the effectiveness of the proposed disturbance attenuating control. Because there is no restriction that the time derivative of the delay is smaller than 1 which is a hypothesis of many articles concerning time-varying delayed systems, the proposed scheme has significance impact on the design and applications of the disturbance attenuating control. Meanwhile an example of CNNs is offered to show the usefulness of the controller to the systems without time delay.

1. Introduction

The well-known cellular neural networks (CNNs), proposed by Chua and Yang [1] in 1988, have been intensively studied theoretically due to their extensive applications in signal and image processing, pattern recognition, parallel computations, as well as optimization problems [2–5]. In many applications due to the finite switching speed of amplifiers in electronic networks or finite speed for signal propagation in biological networks, time-delays inevitably occur which frequently lead to instability and oscillation, bifurcation or chaos. It is well known that the stability of neural network is the prerequisite for its applications in either pattern recognition or optimization solving. Thus, it is very important and significant to investigate the stability of CNN with time-delays (DCNNs) [6–7].

Over the past decades, many researchers have made great efforts to investigate the stability of time-delayed cellular neural networks and many encouraging results have been obtained on the delay-independent or delay-dependent stability analysis for the dynamics systems. The LMI-based techniques have been successfully employed in a variety of stability analysis for neural networks. Asymptotic stability of DCNNs were studied in [8–11]. These papers only concern with stability properties of such systems, without providing any information about the transient responses and decay rates (i.e. exponentially convergence rates) of the system's states. Thus, exponential stability is analyzed for DCNNs. The study on exponential stability and estimation of the exponential convergence rates for neural networks with constant or time-varying delays is carried out in [12–13]. In [14], by applying the Finsler's Lemma and constructing appropriate Lyapunov–Krasovskii functional based on delay partitioning, several improved delay-dependent conditions are developed to estimate the neuron state with some available output measurements such that the error-state system is global asymptotically stable. It sometimes includes

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fewer LMI variables. Compared with the method of Lyapunov functionals as in most previous studies, a new sufficient condition is presented in [15] ensuring the global exponential stability of cellular neural networks with time-varying delays by using an approach based on delay differential inequality combining with Young inequality, which is simpler and more effective for stability analysis. Further the robust asymptotic stability criteria or robust exponential stability criteria are established to cope with parametric uncertainties of the weight coefficients of the neurons such as in [16–22]. A common shortcoming of these contributions is that, though at times robustness against model inaccuracies is considered, little is known on how to deal efficiently with persistent disturbances.

When we deal with neural networks with time-varying delays, a class of systems is unavoidable under persistent disturbance. To the best of our knowledge, however, the disturbance attenuating controller design for DCNNs has not yet been explored. Accordingly, the current study develops a disturbance attenuating control scheme for the closed-loop DCNNs to achieve a bounded attractor for any disturbances belonged to a given set. The plan of the paper is as follows. In Section 2, the model formulation and some preliminaries are given. And the definition of disturbance attenuating controller is presented which gives the main objectives of this paper. Section 3 is devoted to deriving the sufficient conditions for the disturbance attenuating controller for DCNNs. By fully consideration of the four different cases of states in DCNNs and applying Razumikhin theorem, the QMI criterion for getting disturbance attenuating controller is obtained. The proposed controller can be changed into the LMI problem which can be solved efficiently by the help of MATLAB. Finally, in Section 4, numerical examples are presented to show the feasibility and effectiveness of our results.

2. Problem statement and preliminaries

Throughout this paper, $\mathbb{R} = (-\infty, +\infty), \mathbb{R}^n$ denotes any real n -dimensional Euclidean space. Consider the following DCNNs subject to persistent disturbances:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + \sum_{k=1}^{m} d_{ik} h_k(t) + u_i, \quad t \geqslant 0, \\ x_i(t) = \phi_i(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n \quad n \geqslant 2$ is the number of neurons in the network), $c_i > 0$ represents the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs, τ_j represents the transmission delay along the axon of the jth unit and satisfies $0 \leqslant \tau_j(t) \leqslant \tau$; u_j is external input; a_{ij} and b_{ij} are the synaptic connection strengths; $f(x_j(t)) = \frac{1}{2}(|x_j(t)+1| - |x_j(t)-1|), \quad (j = 1, 2, \ldots, n)$ is the activation function of the jth neuron at time t; $h_k(t)$ (k = 1, 2, ..., m) is the bounded disturbance, $\phi_i(t)$ is the initial function, and is assumed to be bounded and continuous on $[-\tau, 0]$.

System (1) can be rewritten into the followed nonlinear differential equation of vector form:

$$\begin{cases} \dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + Dh(t) + u(t), \quad t \geqslant 0 \\ x(t) = \phi(t), \quad t \in [-\tau, 0], \end{cases} \tag{2}$$

where $x(t) = \mathrm{col}\{x_i(t)\} \in \mathbb{R}^n$ is the state vector; $C = \mathrm{diag}\{c_i\}$, $A = (a_{ij})$, $B = (b_{ij}) \in \mathbb{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $D \in \mathbb{R}^{n\times m}$ is the weighting coefficients of the disturbance; $f(x(t)) = col\{f_j(x_j(t))\}$, $f(x(t-\tau(t))) = col\{f_j(x_j(t-\tau_j(t)))\}$, $h(t) = \mathrm{col}\{h_k(t)\} \in \mathbb{R}^m$; $u(t) \in \mathbb{R}^n$ is the control. Here we have slightly abused the notation by using $f(\cdot)$ to denote both the scalar valued and the vector valued functions. In addition, without loss of generality, we assume that the bounded disturbance belongs to the set $\mathbb{H} = \{h \big| h^{\mathrm{T}} h \leqslant 1\}$.

Let the state feedback be $u = -Kx$, then the closed-loop systems of DCNNs (2) is

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + Dh(t) - Kx(t), \tag{3}$$

For an initial state $x(t) = \phi(t)$, $t \in [-\tau, 0]$, we denote the state trajectory of the closed-loop systems (3) under h as $x = x(t, \phi, h)$. A set D in $\mathbb{R}^n$ is said to be invariant if all the trajectories starting from it will remain in it for any $h \in \mathbb{H}$. If the invariant set D also satisfy that any trajectories starting from outside the set D will eventually enter into the set for any $h \in W$, then the set D is called the attractor of the closed-loop systems (3). For the purpose of disturbance rejection, we would like to have a small attractor containing the origin in its interior so that a trajectory will eventually stay close to the origin. Now we formally state the objectives of this paper as follows.

Definition. Given the DCNNs (2), the controller $u = -Kx$ is called disturbance attenuating if the closed-loop systems (3) satisfy the following conditions:

(1) When $h(t) = 0$, the closed-loop systems (3) are globally asymptotically stable;
(2) When $h(t) \neq 0$, there exists a bounded attractor for the closed-loop systems (3).

To construct such controller, let us first review the famous Razumikhin-type theorems for the latter use to deal with delayed differential equations which in mathematics are also called retarded functional differential equations.