Applied Mathematics and Computation 242 (2014) 1–9

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Torsional wave propagation in a self-reinforced medium sandwiched between a rigid layer and a viscoelastic half space under gravity

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ARTICLE INFO

Keywords:
Torsional wave
Self-reinforced
HypergeometricU
LaguerreL
Viscoelastic

ABSTRACT

The present paper constitutes the study of torsional surface wave propagation in a self-reinforced layer resting over a gravitating viscoelastic half space. The layer has an inhomogeneity of linear type associated with the rigidity and density of the medium. Dispersion equation has been obtained in the terms of *HypergeometricU* and *LaguerreL* function. The dispersion equation reduces to a classical form when the inhomogeneity of the layer, gravity and viscosity of the half space vanishes. The influence of various parameters has been depicted by means of graphs for both reinforced and reinforcement free medium.

1. Introduction

As the earth's crust and mantle are not homogeneous, it is interesting to know the propagation pattern of torsional surface wave due to point source in a self-reinforced medium sandwiched between a rigid layer and a viscoelastic half space under gravity. Self-reinforced materials are a family of composite materials, where the polymer fibers are reinforced by highly oriented polymer fibers, derived from the same fiber. Available information suggests that the layered media, crystals and some other materials such as fiber reinforced materials, fluid saturated porous materials etc. exhibits anisotropy. Some hard and soft rocks beneath the earth surface show the reinforcement properties, i.e. the different components acts as a single anisotropic unit as long as they remain in the elastic condition (i.e. the two components are bound together so that there is no relative displacement between them). The Earth's crust contains some hard and soft rocks or materials that may exhibit self-reinforcement properties. These rocks when come in the way of seismic waves do affect their propagation and such seismic signals are always influenced by the elastic properties of the media through which they travel. Alumina or concrete is an example of self-reinforced material. Under certain temperature and pressure some fiber materials may also be modified to self reinforced materials by reinforcing a matrix material of the same fiber.

The study of seismic wave gives important information about the layered earth structure and has been used to accurately determine the earthquake epicenter. Artificially generated seismic waves provide information about the configuration of rock layers for oil exploration and, on a smaller scale, information as to the rigidity of shallow layers for engineering purposes. Thus, modeling of seismic wave propagation plays a significantly important role and is of great utility in the exploration of petroleum, earthquake disaster prevention, civil engineering and signal processing. The study of mechanical behavior of a self-reinforced material has a great importance in geomechanics. Many elastic fiber-reinforced composite materials are strongly anisotropic in behavior. Earthquakes generate waves on the grandest scale, with surface waves observable after

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http://dx.doi.org/10.1016/j.amc.2014.05.010
0096-3003/

several trips around the world, and their systematic study has obvious implications for man's safety, as well as for his curiosity concerning the structure and evolution of the earth. Properties of rocks penetrated by oil wells have been determined by observing seismic waves at various depths, due either to a distant explosion or to a sound source nearby in the same well. Hence, the study of the surface waves and their propagation in various media is of great geophysical significance.

Under certain temperature and pressure, some fiber materials may be modified to self-reinforced material. In real life the fibers might be carbon, nylon, or conceivably metal whiskers. It has been observed that the propagation of elastic surface waves is affected by the elastic properties of the medium, through which they travel (Achenbach [1]). Transmission of Shear waves through a self-reinforced layer sandwiched between two inhomogeneous viscoelastic half-spaces has been discussed by Choudhary et al.[2]. Chaoudhuary et al. [3] has also explained the response of plane SH-wave from elastic slab interposed between two different self -reinforced elastic solids. In 1984, Vardoulakis [4] in his paper showed propagation of torsional surface wave in a homogeneous elastic media. Then several authors worked in this area, few of recent works are Chattoapadhyay et al. [5], Georgiadis et al. [6], Gupta et al. [7], Vishwakarma et al. [8–11] and Gupta et al. [12]. Impulse Response of Elastic Half-Space in the Wave Number-Time Domain has been investigated by Kausel and Park [13] while Green's function for SH-Wave in cylinderically monoclinic material has been explained by Watanabe and Payton [14]. Reference can also be made to Belfield et al. [15] and Spencer [16] for their contribution in the field of elasticity. It's Verma [17] and Verma [18] et al. who described Magnetoelastic Shear Waves and Magnetoelastic Transverse wave in Self reinforced medium, respectively whereas Diaz et al. [19] develop models of elastic transversely isotropic composite using the asymptotic homogenization method.

So far it has been found that the effect of self-reinforcement, gravity and viscosity on the propagation of torsional surface wave in a layer sandwiched between a rigid boundary plane and a half space have remained un-attempted. Therefore, in the present paper attempt has been made to study the propagation of torsional surface waves in a self reinforced layer resting over a gravitating viscoelastic half space. torsional surface wave is a wave with amplitude decaying exponentially with distance from the free surface. Inside the Earth, a very hard layer (also known as "rigid") is present. Since the composition of the Earth is heterogeneous including a very hard layer, the inhomogeneous medium and the rigid interface play significant roles in the propagation of the seismic waves.

2. Formulation and solution of the problem

For the study of torsional surface waves, a cylindrical co-ordinate system has been considered. We considered a self-reinforced layer of thickness H. This is a transversely isotropic and homogeneous medium, which corresponds to the idealized material reinforced by a single family of fibers. Self-reinforced layer is lying over a gravitating viscoelastic half space. In the layer, directional rigidities are μ_T, μ_L along r and z direction, respectively which varies linearly along the depth i.e. $\mu_T = \mu'_T(1 + az)$, $\mu_L = \mu'_L(1 + az)$ where a being an inhomogeneity parameter associated with it, dimension of which is same as that of inverse of length. The density in the layer also vary linearly i.e. $\rho = \rho_0(1 + bz)$, where b is an inhomogeneity parameter with dimension inverse of length. In the half space, μ_1 and ρ_1 are the rigidity and density of the medium, whereas μ' is the viscoelastic parameter. The reinforced layer has been kept sandwiched between the rigid boundary plane and the half space. The writers considered the cylindrical coordinate system, in which origin is taken at the center of the interface, axis r is the direction of wave propagation, and axis z is taken positive downward as shown in Fig. 1.

3. Solution of the layer

The constitutive equation for a transversely isotropic linear elastic material with preferred direction a (Spencer [16]) is

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \tag{1}$$

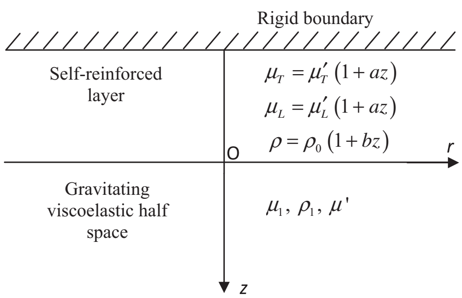

Fig. 1. Geometry of the problem.