Applied Mathematics and Computation 242 (2014) 159–167

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Asymptotic stability analysis of stochastic reaction–diffusion Cohen–Grossberg neural networks with mixed time delays

Yanchao Shi *, Peiyong Zhu

School of Mathematics, University of Electronic Science and Technology of China, Chengdu 611731, China

A R T I C L E I N F O

Keywords:
Asymptotic stability
Cohen–Grossberg neural network
Stochastic system
Lyapunov–Krasovskii functional
Linear matrix inequality

A B S T R A C T

In this paper, the asymptotic stability problem is studied for a class of stochastic Cohen–Grossberg neural networks with reaction–diffusion and time-mixed delays. By using the Lyapunov–Krasovskii functional, stochastic analysis technology and linear matrix inequalities (LMIs) technique, several sufficient conditions on the asymptotic stability for the considered system are obtained. The condition not only connects with the delays and diffusion effect, but also relates to the magnitude of noise. Therefore, these stability criteria are essentially new and more effective than those given in previous conditions. Two examples are presented to illustrate the effectiveness and efficiency of the results.

1. Introduction

Since Cohen–Grossberg neural networks were proposed by Cohen and Grossberg in [1], Cohen–Grossberg neural networks have received much research attention due to their extensive applications in signal processing, image processing, pattern recognition, fault diagnosis, associative memory, combinatorial optimization, and so on. However, time delays are unavoidably encountered due to the finite switching speed of neurons and amplifiers. It has been found that, the existence of time delays may lead to instability and oscillation in a neural network. Therefore, stability analysis of Cohen–Grossberg neural networks with time delays has received much attention, for example, see [2–6] and references therein.

The reaction–diffusion neural networks were firstly introduced by Chua in order to study passivity and complexity in [7]. Strictly speaking, the diffusion effects cannot be ignored in neural networks when electrons move in a nonuniform electromagnetic field [8]. Itoh and Chua had further investigated the complexity of reaction–diffusion neural networks with various boundary conditions in [9]. In 2003, a class of reaction–diffusion Hopfield neural networks with the Neumann boundary conditions was introduced and the global exponential stability of this network was studied [10]. Since then, much works have done on this subject, some results on global asymptotic stability, global exponential stability and periodic solutions for the reaction–diffusion neural networks with various delays and the Neumann boundary conditions, see [11–15] and references therein.

Stochastic effects usually are unavoidable, as Haykin [16] pointed out that the real nervous systems synaptic transmission is noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. In recent year, many interesting results on the stability of stochastic neural networks with delays have been reported, see, e.g., [17–20] and the references therein.

* Corresponding author.
E-mail address: cdycshi@163.com (Y. Shi).

http://dx.doi.org/10.1016/j.amc.2014.05.056

Therefore, it is necessary to consider stochastic effect, diffusion effect and delay effect on the stability of neural networks. However, in [21,22] all criteria on stability for reaction–diffusion neural networks were independent of the diffusion effect. In other words, we do not know the reaction–diffusion phenomenon how to affect stability of neural networks. Recently, in [23,24], the authors proposed some reaction–diffusion delayed neural networks with Neumann boundary conditions and Dirichlet boundary conditions, respectively, and obtained some diffusion-dependent criteria on stability for the formulated neural networks. These criteria show that the diffusion phenomenon is beneficial to the stabilization of neural systems.

As we know, LMIs can be easily checked by using the MATLAB LMI Control Toolbox. However, to the best of our knowledge, few LMI-based stability results have investigated stochastic Cohen–Grossberg neural networks with mixed time-varying delays, reaction–diffusion terms and the Dirichlet boundary conditions. Inspired by the above discussions, the objective of this paper is to study the asymptotic stability for a class of stochastic Cohen–Grossberg neural networks with mixed time-varying delays and diffusion. By using the Lyapunov–Krasovskii functional, stochastic analysis technology [25], Ito's formula and LMIs technique, several sufficient conditions on the asymptotic stability for the considered system are obtained. These results extend and improve the earlier publications.

Throughout this paper, R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T denotes the transpose of a matrix or vector; $tr(\cdot)$ denotes the trace of the corresponding matrix and I denotes the identity matrix. λ_{max} and λ_{min} denote the maximum and the minimum eigenvalues of a real symmetric matrix. $\|\cdot\|$ stands for the Euclidean norm. $diag(\cdot)$ stands for the diagonal matrix. $*$ represents the elements below the main diagonal of a symmetric matrix. $P>0$ means that is a real symmetric positive definite matrix. Define the norm $\|y(t,x)\|_2=\left(\sum_{i=1}^n\|y_i(t,x)\|^2\right)^{1/2}$, where $\|y_i(t,x)\|=\left(\int_S|y(t,x)|^2dx\right)^{1/2}$ for any $y(t,x)\in L^2(R\times S,R^n)$. The noise perturbation σ is the noise intensity matrix; $\omega(t)=(\omega_1,\omega_2,\dots,\omega_n,)$ is an n-dimensional Brownian motions defined on a complete probability space $(\Omega,\mathcal{F},P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}(\mathcal{F}_t=\sigma\{\omega(s):0\leqslant s\leqslant t\})$. Take $\tau=\max\{\tau_1(t),\tau_2(t)\}$ and $C([-\tau,0];R^n)$ denotes the family of continuous function ϕ from $[-\tau,0]$ to R^n with the uniform norm $\|\phi\|=\sup_{-\tau\leqslant s\leqslant 0}|\phi(s)|$. $(\Omega,\mathcal{F},P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e. the filtration contains all P-null sets and is right continuous). Denote by $L^2_{\mathcal{F}_0}([-\tau,0];R^n)$ the family of all $\mathcal{F}_t$ measurable, $C([-\tau,0];R^n)$ valued stochastic variables $\phi=\{\phi(s,x):-\tau\leqslant s\leqslant 0\}$ such that $\int_{-\tau}^0E|\phi(s)|^2ds<\infty$, where $E[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure P.

2. Problem formulation and preliminaries

Consider the stochastic Cohen–Grossberg neural networks with mixed time delays and reaction–diffusion terms described by the following differential equation:

$$\begin{aligned} dy_i(t,x)=&\left\{\sum_{k=1}^m\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k}\right)-\alpha_i(y_i(t,x))\left[\beta_i(y_i(t,x))-\sum_{j=1}^n a_{ij}f_j(y_j(t,x))-\sum_{j=1}^n b_{ij}g_j(y_j(t-\tau_1(t),x))-\sum_{j=1}^n c_{ij}\int_{t-\tau_2(t)}^t h_j(y_j(s,x))ds\right]\right\}dt\\ &+\sum_{j=1}^n\sigma_{ij}(y_j(t,x),y_j(t-\tau_1(t),x),y_j(t-\tau_2(t),x))d\omega(t), \end{aligned} \tag{1}$$

or, in a compact form:

$$\begin{aligned} dy(t,x)=&\left\{\sum_{k=1}^m\frac{\partial}{\partial x_k}\left(D_k\frac{\partial y(t,x)}{\partial x_k}\right)-\alpha(y(t,x))\left[\beta(y(t,x))-Af(y(t,x))-Bg(y(t-\tau_1(t),x))-C\int_{t-\tau_2(t)}^t h(y(s,x))ds\right]\right\}dt\\ &+\sigma(y(t,x),y(t-\tau_1(t),x),y(t-\tau_2(t),x))d\omega(t), \end{aligned} \tag{2}$$

where $A=(a_{ij})_{n\times n},B=(b_{ij})_{n\times n}$ and $C=(c_{ij})_{n\times n}$ are the connection weight strengths. $x=(x_1,x_2,\dots,x_m)^T\in S\subset R^m$, $S=\{x||x_k|\leqslant L_k\},L_k$ is a constant $(k=1,2,\dots,m)$. $y(t,x)=(y_1(t,x),y_2(t,x),\dots,y_n(t,x))^T$, $y_i(t,x)$ is the state variable of the ith neuron at time t and in space variable x. $\alpha(\cdot)$ represents the amplification function; $\beta(\cdot)$ is the behavior function; $f(y(t,x))=(f_1(y_1(t,x)),f_2(y_2(t,x)),\dots,f_n(y_n(t,x)))^T$, $f_j(y_j(t,x)),g_j(y_j(t,x))$ and $h_j(y_j(t,x))$ denote the activation functions of the jth unit at time t and in space variable x. The smooth function $D_k=diag(D_{1k},D_{2k},\dots,D_{nk}),D_{ik}\geqslant 0$ is a diffusion operator, S is a compact set with a smooth boundary ∂S of class C^2 and measure mes $S>0$ in R^m. $\tau_1(t),\tau_2(t)$ are the time delays.

Throughout this paper, we make the following assumptions.

Assumption 1. There exist positive constants $\alpha_i^0,\alpha_i^1(i=1,2,\dots,n)$ such that

$$0<\alpha_i^0\leqslant\alpha_i(y_i(t,x))\leqslant\alpha_i^1$$

for all $x\in R,i=1,2,\dots,n$.

Assumption 2. There exist positive constants $\mu_i(i=1,2,\dots,n)$ such that