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Robust solutions to multi-facility Weber location problem under interval and ellipsoidal uncertainty

Ali Jamalian, Maziar Salahi *

Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran

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ABSTRACT

In this paper, we consider the multi-facility Weber location problem (MFWP) with uncertain location of demand points and transportation cost parameters. Equivalent formulations of its robust counterparts for both the Euclidean and block norms and interval and ellipsoidal uncertainty sets are given as conic linear optimization problems.

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1. Introduction

Location analysis and location decisions are major applications area between operations research and computer science. There exist different kinds of facility location problems [1–6], among them, multi-facility Weber location problem (MFWP) is a well-known problem in operations research and especially in location analysis which has been studied in depth [1,4]. The MFWP is concerned with locating a set of facilities and allocating their capacities to satisfy the demands of a set of customers with known locations so that the total transportation costs is minimized. In this problem, plants and warehouses may constitute the facilities, while retailers may be considered as customers. Efficient algorithms are developed to solve this problem in the literature, for example see, [1,6–8].

Many real-world optimization problems involve data that are noisy or uncertain. So addressing data uncertainty in mathematical programming models is recognized as a central problem in optimization. In recent years, a body of literature is developing under the name of robust optimization which is based on a description of uncertainty sets. The uncertain parameters are only known to belong to known sets, and one associates with the uncertain problem its robust counterpart, where the constraints are enforced for every possible value of the parameters within their prescribed sets; under such constraints, the worst-case value of the cost function is then minimized to obtain a robust solution of the problem [6,9–20].

Zhang and Wu [21] have considered uncertainty on Euclidean single facility location problems, which is a special case of MFWP with $m = 1$, using robust optimization methodology. They have shown this problem with unknown but bounded data or with an ellipsoidal uncertainty set is equivalent to a second-order cone programming (SOCP) and an semidefinite programming (SDP), respectively. Also, they have shown that this problem with ellipsoidal uncertainty set or with box uncertainty is NP-hard and have proposed an explicit SDP to approximate the problem.

The block norms, $\|.\|_B$, are those whose contours are polytopes with extreme points $b_g \in \mathbb{R}^2,\ g = 1, \ldots, r$. The class of block norms is a very general one and comprises for example the l_1 and l_∞ norms. The first application of block norms to solve location problems suggested by Ward and Wendell [22,23]. They have shown that the minimax and minisum single facility location problems with block norms can be written as an LP.

* Corresponding author.
E-mail address: salahim@guilan.ac.ir (M. Salahi).

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One of the most important applications of location problems with block norms is in solving location problems with L_p norm as distance measure. In this case, the problem can be solved to an arbitrary level of approximation of L_p norm with suitable block norm. More importantly, the problem with block norm can be formulated as LP. Since rectilinear and infinity norms are special cases of block norms, thus our results can be extended to these cases as well. In the following, we state modeling of the problem. Let us first define some notations:

m: number of new facilities
n: number of existing facilities
w_{ji}: nonnegative weight between new facility j and existing facility i by a unit distance
v_{jk}: nonnegative weight between the location of new facilities j and k by a unit distance
$d(X_j, P_i)$: distance between the location of new facility j and existing facility i
$d(X_j, X_k)$: distance between the location of new facilities j and k
$P_i : (a_i, b_i)$ the location coordinates of existing facility i
$X_j : (x_j, y_j)$ the location coordinates of existing facility j.

Let m existing facilities be located at the known distinct points $P_1, \ldots, P_m$, in the plane. In MFWP, the optimal location of n new facilities, $X_1, \ldots, X_n$, is sought with respect to the set of existing facilities. Let the cost per unit distance between new facility j and existing facility i be denoted by w_{ji} and v_{jk} being the corresponding cost per unit distance between new facilities j and k. The total transportation cost associated with new facilities located at $X_1, \ldots, X_n$, is given by

$$f(X_1, \ldots, X_n) = \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(X_j, P_i) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(X_j, X_k). \tag{1}$$

The MFWP can be stated as the selection of locations $X_1^*, \ldots, X_n^*$ of new facilities such that total cost in (1) is minimized [2–5].

In this paper, we present robust formulations for locating multiple new facilities with respect to multiple existing facilities and demand points with uncertain data, in two cases, with block and Euclidean norms. The rest of the paper is organized as follows. In Section 2, we study MFWP with block norms and give its robust counterparts for both interval and ellipsoidal uncertainty sets. In Section 3, the robust counterparts of the MFWP with Euclidean norm is given. Finally, we present some conclusions and future research directions.

2. MFWP with block norms

Suppose distances are measured by a block norm B. The MFWP with block norm is as follows:

$$\min \quad \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} \|X_j - P_i\|_B + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} \|X_j - X_k\|_B. \tag{2}$$

For $i = 1, \ldots, m, \quad j = 1, \ldots, n, \quad k = 1, \ldots, n$, let

$$\|X_j - P_i\|_B = \sum_{g=1}^{r} (\beta_{gji}^{+} + \beta_{gji}^{-}), \tag{3}$$

$$X_j - P_i = \sum_{g=1}^{r} (\beta_{gji}^{+} - \beta_{gji}^{-}) b_g,$$

$$\|X_j - X_k\|_B = \sum_{g=1}^{r} (\lambda_{gjk}^{+} + \lambda_{gjk}^{-}), \tag{4}$$

$$X_j - X_k = \sum_{g=1}^{r} (\lambda_{gjk}^{+} - \lambda_{gjk}^{-}) b_g,$$

where λ_{gjk}^{+}, λ_{gjk}^{-}, β_{gji}^{+}, β_{gji}^{-}'s are nonnegative variables and b_g's are extreme points of polytope corresponding to unit contour of the block norm $\|.\|_B$. Using these transformations, (3) and (4) becomes the following LP:

$$\begin{aligned}
\min \quad & Z = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{g=1}^{r} w_{ji}(\beta_{gji}^{+} + \beta_{gji}^{-}) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{g=1}^{r} v_{jk}(\lambda_{gjk}^{+} + \lambda_{gjk}^{-}) \\
s.t. \quad & X_j - X_k = \sum_{g=1}^{r} (\lambda_{gjk}^{+} - \lambda_{gjk}^{-}) b_g, \quad j = 1, \ldots, n, \quad k = 1, \ldots, n, \\
& X_j - P_i = \sum_{g=1}^{r} (\beta_{gji}^{+} - \beta_{gji}^{-}) b_g, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \\
& \lambda_{gjk}^{+}, \lambda_{gjk}^{-}, \beta_{gji}^{+}, \beta_{gji}^{-} \geqslant 0, \quad \begin{array}{l} g = 1, \ldots, r, \quad i = 1, \ldots, m, \\ j = 1, \ldots, n, \quad k = 1, \ldots, n. \end{array}
\end{aligned} \tag{5}$$

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