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Approximate controllability of semi-linear neutral integro-differential systems with finite delay ☆

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ABSTRACT

This paper considers the approximate controllability of semilinear neutral integrodifferential systems with finite delay in Hilbert space. Since the considered equation admits nonlinear terms involving spatial derivatives, the fraction power theory and α-norm are used to discuss the problem so that the established results can be applied to them. The results are obtained by using the Sadovskii fixed point theorem and the theory of analytic resolvent operator. An example is presented to illustrate the applications of the obtained results.

1. Introduction

In this paper, we consider the approximate controllability of systems represented in the following semi-linear neutral integro-differential equations with finite delay

$$\begin{cases} \frac{d}{dt}[x(t) + F(t, x_t)] = -Ax(t) + \int_0^t \gamma(t-s)x(s)ds + G(t, x_t) + Bu(t), & t \in [0, T], \\ x_0 = \phi, & t \in [-r, 0], \end{cases} \tag{1}$$

where the state variable $x(\cdot)$ takes values in a Hilbert space X and the control function $u(\cdot)$ is given in the Banach space $L^2([0,T];U)$ of admissible control functions, U is also a Hilbert space. The (unbounded) linear operator $-A$ generates an analytic semigroup on X. $\gamma(\cdot)$ is a family of closed linear operators to be defined later. B is a bounded linear operator from U into X. $F : [0,T] \times C_\alpha \longrightarrow X_{\alpha+\beta}$ and $G : [0,T] \times C_\alpha \longrightarrow X$ are appropriate functions to be specified below. $\phi \in C_\alpha := C([-r,0];X_\alpha),\ X_\alpha \subset X$ is a Banach space to be described below and, for any function $x(\cdot) \in C([0,T];X)$, the histories x_t are defined in the usual way by $x_t(\theta) = x(t+\theta)$ for $\theta \in [-r, 0]$.

Controllability theory for abstract linear control systems in infinite-dimensional spaces is well-developed, and the details can be found in various papers and monographs, see [6] and references therein. Several authors have extended these concepts to infinite-dimensional systems represented by nonlinear evolution equations, see [24,25,37]. Most of the controllability results for nonlinear infinite-dimensional control systems concern the so-called semi-linear control systems that consist of a linear part and a nonlinear part. Zhou [37] studied the approximate controllability of an abstract semi-linear control system by assuming certain inequality conditions that are dependent on the properties of the system components. Naito [24,25] studied the approximate controllability of the same system. He showed that under a range condition on the control action operator, the semi-linear control system is approximately controllable if the corresponding linear system is so. Bashirov and

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Mahmudov showed in paper [3] that under an appropriate condition on a resolvent operator the approximate controllability of semi-linear systems is implied by the approximate controllability of its linear part. This resolvent condition is convenient for applications and it has been used in many papers to study the approximate controllability for nonlinear (fractional order, functional) differential equations, see, for instance, [7,9,10,30,31].

Integro-differential equations can be used to describe a lot of natural phenomena arising from many fields such as electronics, fluid dynamics, biological models, and chemical kinetics. Most of these phenomena can not be described through classical differential equations. That is why in recent years they have attracted more and more attention of several mathematicians, physicists, and engineers. Some topics for this kind of equations, such as existence and regularity, stability, (almost) periodicity of solutions and control problems, have been investigated by many mathematicians, see [1,17,18,20–22,34,36], for example.

In [14–16], Grimmer et al. proved the existence of solutions of the following integro-differential evolution equation

$$\begin{cases} v'(t) = Av(t) + \int_0^t \gamma(t-s)v(s)ds + g(t), & \text{for } t \geqslant 0, \\ v(0) = v_0 \in X, \end{cases} \tag{2}$$

where $g : \mathbb{R}^+ \to X$ is a continuous function. The author(s) showed the existence, uniqueness and the representation of solutions for (2) via resolvent operators associated to the following linear homogeneous equation

$$\begin{cases} v'(t) = Av(t) + \int_0^t \gamma(t-s)v(s)ds, & \text{for } t \geqslant 0, \\ v(0) = v_0 \in X. \end{cases}$$

That is, the resolvent operator replacing the role of C_0-semigroup for evolution equations, plays an important role in solving Eq. (2) in weak and strict senses. In recent years much work on existence problems for nonlinear integro-differential evolution equations has been done by many authors through applying the theory of resolvent operator, see Ref. [8] and the references therein.

Particularly, (impulsive) neutral (integro) differential equations arise in many areas of applied mathematics. For instance, the system of rigid heat conduction with finite wave speeds, studied in [13], can be modeled in the form of integro-differential equations of neutral type with delay, and for this reason these equations have received much attention in the last few years, see [4,5,11,12,19].

The controllability problem of neutral integro-differential systems is naturally an important and interesting topic for mathematicians, and meanwhile it is also a difficult problem. Many papers, see Paper [2,32] among others, have studied by using semigroup methods the approximate controllability problem for integro-differential systems for which $\gamma(\cdot) \equiv 0$ and $G(\cdot,\cdot)$ involves an integral term. For the controllability of the neutral system (1) with $\gamma(\cdot) \neq 0$, the situation is much more complicated and little is known to the best of our knowledge. In paper [33], the authors have studied the approximate controllability of the following neutral integro-differential equations with impulses and unbounded delay

$$\begin{cases} \frac{d}{dt}D(t,x_t) = AD(t,x_t) + \int_0^t \gamma(t-s)D(t,x_t)ds + f(t,x_t) + Bu(t), & t \in J = [0,b], \\ x_0 = \phi \in \mathbb{P}, \\ \Delta x(t_k) = I_k(x-t-k), k = 1,\ldots,m, \end{cases} \tag{3}$$

where $D(t,\phi) = \phi(0) + g(t,\phi)$ and $g : J \times \mathbb{P} \to X$ is an appropriate function. They achieved the controllability result for (3) by imposing compactness assumptions on the resolvent operator $W(t)$ and on the operator semigroup generated by A, they also assumed that the corresponding linear system of (3) is approximately controllable.

The aim of the present work is to investigate the approximate controllability for the neutral system (1) with the aid of the resolvent operator theory. Since in many practical models the nonlinear terms involve frequently spacial derivatives, in this case, we can not discuss the problem on the whole space X (it is often taken as $X = L^2([0,\pi])$) because the history variables of the functions F and G are only defined on $C([-r,0];X_{\frac{1}{2}})$ rather than $C([0,T];X)$. Hence, the discussion of [33] becomes invalid. In order to make the results be valid for this kind of (more general) systems, we shall study the problem by applying the theory of fractional power operators and $\alpha-$norm. That is, we suppose that the operator $(-A, D(-A))$ generates a compact analytic semigroup on X so that the resolvent operator $W(t)$ is analytic as well and the space $X_\alpha(\subset X)$ is well defined. In this way, we first restrict this system in the Hilbert space X_α to prove the existence of mild solutions via $\|\cdot\|_\alpha$ and the fractional power theory, then we investigate the controllability for system (1) in space X.

We point out here that we do not require that the resolvent operator $W(t)$ be compact for $t > 0$ which differs greatly from that in [33] (note that the compactness of the semigroup generated by $-A$ does not imply that $W(t)$ is compact). As for the uniform continuity of $W(t)$, it is guaranteed by the anality (from [12], Lemma 2.3). One should also note that, even the resolvent operator $W(t)$ is compact, it is not necessarily uniformly continuous since it has not the semigroup property. In addition, generally speaking, it is difficult to get the explicit formula of the resolvent operator $W(t)$, hence, one can not easily check the approximate controllability of the corresponding linear system of (1), which is the main assumption (see the condition (H_0) in Section 2) of this paper as well as others on this topic. In fact, in the discussion of the example in Paper [33], the authors mentioned that the approximate controllability of the corresponding linear system was guaranteed by the compactness of the resolvent operator $W(t)$. This, however, is not clear to us. In Section 4 we go through this difficulty successfully and verify this hypotheses exactly by proving the self-adjoint property of operator $W(t)$ (see Lemma 4.1).