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Meshless local B-spline-FD method and its application for 2D heat conduction problems with spatially varying thermal conductivity

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ABSTRACT

In this paper, a new class of meshless methods based on local collocation and B-spline basis functions is presented for solving elliptic problems. The proposed approach is called as meshless local B-spline basis functions based finite difference (local B-FD) method. The method was straightforward to develop and program as it was truly meshless. Only scattered nodal distribution was required hence avoiding at all mesh connectivity for field variable approximation and integration. In the method, any governing equations were discretized by B-spline approximation in the spirit of FD technique using local B-spline collocation i.e. any derivative at a point or node was stated as neighboring nodal values based on the B-spline interpolants. In addition, as B-spline basis functions pose favorable properties such as (i) easy to construct to any arbitrary order/degree, (ii) have partition of unity property, and (iii) can be easily designed to pose the Kronecker delta property, the shape function construction as well as the imposition of boundary conditions can be incorporated efficiently in the present method. The applicability and capability of the present local B-FD method were demonstrated through several heat conduction problems with heat generation and spatially varying conductivity.

1. Introduction

The analysis of heat transfer problem in complex domains is very important to engineering and science and can be found in many technological applications such as electronic cooling, thermal insulation or heat conduction in triangular or circular fins attached to a pipe of vapor transport. Traditional mesh-based methods such as finite difference (FD), finite element (FE) and finite volume (FV) methods are commonly employed to solve the heat conduction problems. Nevertheless, due to the fact that mesh generation can be very time consuming and is the most expensive part of a simulation [1], large attention has been given in recent years to the so-called meshfree methods as new emerging numerical techniques in engineering and science.

The interests for meshless methods may be traced back to the introduction of smoothed particle hydrodynamics (SPH) method by Lucy [2] and Gingold and Monaghan [3], and diffused element method (DEM) by Nayroles et al. [4]. A number

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of meshless methods have been developed since then. Belytschko et al. [5] proposed element-free Galerkin (EFG) method. Liu et al. [6] introduced reproducing kernel particle method (RKPM). Meshless local Petrov–Galerkin (MLPG) method was introduced by Atluri and Zhu [7,8] and Atluri and Shen [9]. The method was the basis for several other meshless methods. In addition, radial basis functions (RBF) collocation method, pioneered by Kansa [10], is getting more interest in research community due to its simplicity and flexibility. Local RBF collocation methods have been developed by Shu et al. [11], Tolstykh and Shirobokov [12] and also further applications of the RBF based FD methods in [13,14]. It is also noted here, Le et al. [15] has proposed a collocation method based on one-dimensional RBF interpolation scheme for solving PDEs. Moreover, Liu et al. [16] and Liu [17] proposed point interpolation methods for computational mechanics. It is worthy to also note here the advancement of the radial point interpolation method (RPIM) developed in [16,17] in the field of computational electromagnetics for stable meshless time-domain modeling by Yu and Chen [18–20], Kaufmann et al. [21,22] and Kaufmann and Engstrom [23]. Furthermore, an improved SPH method, called as smoothed particle electromagnetic method, has been employed by Ala and Francomano for analyzing transient electromagnetic phenomena [24,25], non-invasive investigations of neuronal human brain activity [26] and by Ala et al. [27] for solving the magnetoencephalography forward problem.

In the field of heat transfer, implementations of meshless methods have been reported in literature. A meshless model for transient heat conduction in FGM was presented by Wang et al. [28]. Gao [29] employed a meshless BEM for isotropic heat conduction problems with heat generation and spatially varying conductivity. MLPG method for 2D steady-state heat conduction problems of irregular domains was investigated by Wu et al. [30]. Meshless EFG method for nonlinear heat conduction problems was presented by Singh et al. [31]. Singh and Tanaka [32], Sladek et al. [33] and Li et al. [34] presented heat transfer analyses in 3D applications. Li et al. [35] employed the MLPG method for transient heat conduction analysis with modified precise time step integration method. In a separate study, Chen and Liew [36] presented local Kriging interpolation for transient heat conduction problems, where Soleimani et al. [37] employed the RBF-DQ method for 2D transient conduction problems in complex geometries. In [38], numerical solution of transient heat conduction problems using improved meshless local Petrov–Galerkin method was presented by Dai et al.

Despite the continuous progress in the development and application of meshless methods, main numerical concerns still remain. Meshless shape functions construction is more complicated and time consuming than that for FEM. Most of meshless shape functions lack of the Kronecker delta function property, which lead to difficult and tedious imposition of boundary conditions. In general, the efficiency of meshless methods is still inferior to that of FEM/FDM due to the complicated processes of the shape function construction and the implementation of boundary conditions. Several approaches have been introduced to further increase the robustness of meshless methods. For an instance, Wen and Aliabadi [39] introduced indirect technique for MLS shape function's high order derivatives based upon the first order derivative. Coupling of the meshfree and finite element methods for determination of the crack tip fields was presented by Gu and Zhang [40]. Zhang et al. [41], in their attempt, introduced the use of arbitrary convex polygon for nodal influence domain in EFG method rather than circle and rectangle nodal influence domains commonly adopted. Extending the concept of moving least-squares approximation, Ren et al. [42] introduced the complex variable interpolating moving least-squares method. Moreover, there has been an increasing interest recently in the search of basis functions that would be more favorable than the functions commonly used in the meshless methods. For examples, Kriging interpolation has been examined in [36,38].

In this paper, a new class of meshless methods based on local collocation and B-spline basis functions is presented for solving elliptic problems. The proposed approach is called as meshless local B-spline basis functions based finite difference (local B-FD) method. Our main motivation and objective for the present work is to develop a B-spline based meshless method which poses high flexibility in dealing with domains of arbitrary geometries. It is well known that B-spline based numerical methods, in particular B-spline collocation methods, have been recognized as successful methods enjoying spectral or exponential convergence for applications with rectangular or regular domains. However, study on effective and efficient B-spline based methods for applications with complex domains is challenging and still an open research area. As stated by Sun et al. [43], the study on efficient spline collocation methods for solving PDEs in a complex region has been an attractive area, but it seemed not very successful. It can be noted here that although several attempts have been devoted to develop spline based methods which are suitable for the analysis in complex domains, the potential of B-spline basis functions incorporated in the framework of generalized FD method has not been yet intensively explored and exploited. For instances, the procedures of implanting and mapping the complex domains into a rectangle have been previously employed by Cooper [44] and Van Blerk and Botha [45], while the Galerkin method coupled with weighted extended tensor product B-splines has been used by Höllig et al. [46,47].

It is worth to note that the idea of the FD discretization which is valid in the neighborhood of a given point can be used to construct a discretization scheme using random nodal distributions. The framework hence seems to be a natural choice for such a pursuit. In meshless method implementation, the framework of generalized FD method has been also proven as a successful way to solve the conditioning problem in the collocation methods with global RBFs [11–14].

In the proposed method, any governing equations were discretized by B-spline approximation in the spirit of FD technique using local B-spline collocation i.e. any derivative at a point or node was stated as neighboring nodal values based on the B-spline interpolants. This was the key aspect of the present local B-FD method. As only scattered nodal distribution was required, the method was truly meshless thus avoiding at all mesh connectivity for field variable approximation and integration. This made the method straightforward to develop and program. Moreover, as B-spline basis functions pose favorable properties such as (i) easy to construct to any arbitrary order/degree, (ii) have partition of unity property, and