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Stochastic finite-time boundedness for Markovian jumping neural networks with time-varying delays

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ABSTRACT

In this paper, a novel method is developed for the finite-time boundedness of Markovian jumping neural networks with time-varying delays. By introducing a newly augmented stochastic Lyapunov–Krasovskii functional and novel activation function conditions, sufficient condition for Markovian jumping neural networks is presented, and the state trajectory remains in a bounded region over a pre-specified finite-time interval. Finally, numerical examples are given to illustrate the efficiency and less conservative of the proposed method.

1. Introduction

Over the past several decades, neural networks have attracted considerable attention due to their potential applications in various fields such as signal processing, pattern recognition and other scientific fields [1–4]. In many of these practical applications, it is always required that the neural networks has the equilibria point. Time delay is always encountered since the neural networks are frequently implemented by all kinds of hardware circuits-digital or integrated circuits. It is well known that time delay is the source of chaos and instability. Therefore, it is very important to study the stability of neural networks with time delay. Up to now, a great number of researchers have devoted effort and time to the stability analysis of delayed neural networks [5–10].

It is worth pointing out that, at the different times, a neural network with finite modes frequently switch from one to another. For neural networks with time delay, it has been demonstrated that the switching between the different neural network modes can be decided by a Markov process. Therefore, Markovian jump systems (MJSs) have received a large mount of research attention, for instance, sudden environmental disturbance, component failures and abrupt variations [11–16]. In the literature, Markovian jump neural networks are of major important in modeling both continuous-time and discrete-time neural networks [17–26]. It should be noted that, most of the above results consider an infinite-time interval only.

In reality, we are interested in the behavior of the neural networks over some finite time interval. For example, large values of the states are not acceptable in the presence of saturations. Once we fix the finite time interval, the state of neural networks does not exceed a certain bound during the specified time interval. Therefore, in finite-time interval, finite-time stability is investigated to address these transient performances of control systems. Recently, the concept of finite-time

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stability has been revisited in terms of linear matrix inequalities (LMIs) and Lyapunov function theory, some results are obtained to ensure that system is finite-time stability or finite-time boundedness [27–35]. To the best of our knowledge, the finite-time stability analysis for a class of Markovian jumping neural networks with mode-dependent time-varying delays has not been fully tackled except for [29,30]. It should be mentioned that in [29,30], the stability criteria of neural networks is conservative. There still exists room for further improvement because some useful terms are ignored in the stochastic Lyapunov–Krasovskii functional employed in [29,30]. It is natural to look for an alternative view to reduce the conservatism of stability criteria. This has motivated our research on this issue.

In this paper, finite-time boundedness for a class of Markovian jumping neural networks with time-varying delays is investigated. The main contributions of this paper are given as follows. First, paying more attention to activation functions and a newly augmented stochastic Lyapunov–Krasovskii functional is proposed. Cross terms among the states are employed with the condition of $\gamma_i^- \leqslant \frac{g_i(\varsigma_1)-g_i(\varsigma_2)}{\varsigma_1-\varsigma_2} \leqslant \frac{\gamma_i^-+\gamma_i^+}{2}$ and $\frac{\gamma_i^-+\gamma_i^+}{2} \leqslant \frac{g_i(\varsigma_1)-g_i(\varsigma_2)}{\varsigma_1-\varsigma_2} \leqslant \gamma_i^+$. Second, triple-integral terms, four-integral term are introduced in this paper and distinct Lyapunov matrices $(P_{r_t}, \mathcal{Q}_{1r_t}, \mathcal{Q}_{2r_t}, S_{r_t}, X_{r_t}, Z_{r_t})$ are chosen for different system modes to reduce the conservatism the stability criterion. Third, finite-time stability is an independent concept from Lyapunov stability and can be affected by switching behavior significantly. There exists nonlinear parameters and time-varying delays in the stochastic dynamic Markovian jumping neural networks, sufficient stability condition with respect to the finite-time interval is given.

The remainder of this paper is organized as follows: In Section 2, some basic definitions and preliminary lemmas are introduced. Section 3 discusses the sufficient condition that guarantee stochastically finite-time boundedness of Markovian jump systems with time-varying delays. Then, numerical examples are given to illustrate the efficiency of proposed technique in Section 4. Finally, conclusion is stated.

2. Preliminaries

Given a probability space (Ω, F, P) where Ω, F and P respectively represent the sample space, the algebra of events and the probability measure defined on Ω. In this paper, we consider the following n-neuron Markovian jumping neural network over the space (Ω, F, P) described by

$$\begin{cases} \dot{x}(t) = -A_{r_t}x(t) + B_{r_t}f(x(t)) + C_{r_t}f(x(t-\tau_{r_t}(t))) + J \\ x(t) = \phi(t), \quad t \in [-\tau, 0), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^\top$ represents the neural state vector of the system,

$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^\top$ is the nonlinear activation function with the initial condition $f(0) = 0$; $A_{r_t} = \mathrm{diag}\{a_1(r_t), a_2(r_t), \ldots, a_n(r_t)\}$ describes the rate with each neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $B_{r_t} = [b_{ij}(r_t)]_{n\times n}$ and $C_{r_t} = [c_{ij}(r_t)]$ are the connection weight matrix and the delayed connection weight matrix, respectively; $J = [J_1, J_2, \ldots, J_n]^\top$ denotes a constant external input vector. $\tau_{r_t}(t)$ is the time-varying delay which satisfies

$$0 \leqslant \tau_{r_t}(t) \leqslant \tau_{r_t}, \tag{2}$$

$$h_{r_t} \leqslant \dot{\tau}_{r_t}(t) \leqslant d_{r_t} < 1. \tag{3}$$

where τ_{r_t} and d_{r_t} are constant scalars, and $\tau = \max_{r_t}\{\tau_{r_t}\}, d = \max_{r_t}\{d_{r_t}\}, h = \min_{r_t}\{h_{r_t}\}$.

There exists a parameter $0 \leqslant \chi \leqslant 1$ such that $\dot{\tau}_{r_t}(t)$ can be expressed as a convex combinations of the vertices

$$\dot{\tau}_{r_t}(t) = \chi h_{r_t} + (1-\chi)d_{r_t}. \tag{4}$$

Remark 1. It should be noted that, if a stability condition is dependent on $\dot{\tau}_{r_t}(t)$, it needs to check at the vertex values of $\dot{\tau}_{r_t}(t)$ instead of checking all values of $\dot{\tau}_{r_t}(t)$. This technique is employed in this paper to reduce the conservatism of stability criteria for systems with time-varying delays.

Let the random form process $\{r_t, t \geqslant 0\}$ be the Markov stochastic process taking values on a finite set $\mathcal{N} = \{1, 2, \ldots, N\}$ with transition rate matrix $\Omega = \{\pi_{ij}\}, i, j \in \mathcal{N}$, namely, for $r_t = i$, $r_{t+1} = j$, one has

$$\Pr(r_{t+h} = j | r_t = i) = \begin{cases} \pi_{ij}h + o(h), & \text{if } j \neq i \\ 1 + \pi_{ii}h + o(h), & \text{if } j = i \end{cases}$$

where $h > 0, \lim_{h\to 0}\frac{o(h)}{h} = 0$ and $\mu \geqslant 0$ $(i, j \in \mathcal{N}, j \neq i)$ denotes switching rate from mode i at time t to mode j at time $t+h$. For all $i \in \mathcal{N}$, $\pi_{ii} = -\sum_{j=1, j\neq i}\pi_{ij}$.

Set $\mathcal{N}$ contains N modes of system (1) and for $r_t = i \in \mathcal{N}$, the system matrices of the ith mode are denoted by A_i, B_i and C_i, which are considered to be real known with appropriate dimensions.

Assumption 1. The neuron state-based nonlinear function $f(x(t))$ in system (1) is bounded and satisfies: