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Single-machine bicriterion group scheduling with deteriorating setup times and job processing times

Jian-Jun Wang *, Ya-Jing Liu

Faculty of Management and Economics, Dalian University of Technology, Dalian 116024, PR China

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ABSTRACT

This paper considers a group scheduling problem with two ordered criteria where both setup times and job-processing times are increasing functions of their starting times. It is assumed that the jobs be classified into several groups and the jobs of the same group have to be processed contiguously. We consider two objectives where the primary criterion is the total weighted completion time and the secondary criterion is the maximum cost. A polynomial time algorithm is presented to solve this bicriterion group scheduling problem with deteriorating setup times and job-processing times. This algorithm can also solve single-machine group scheduling problems with deteriorating setup times and job-processing times in several ordered maximum cost and arbitrary precedence.

1. Introduction

In classical group scheduling problems, most research assumes that the processing time of a job and the setup time of a group be a constant. However, there are many situations in which a job or a group that is processed later consumes more time than the same job or group when processed earlier. For example, in fire fighting when the time and effort required to control a fire increases if there is a delay in the start of the fire-fighting effort. Hence, there is a growing interest in the literature to study scheduling problems involving deteriorating jobs, i.e. jobs whose processing times are increasing functions of their starting times. We refer the reader to the book by Gawiejnowicz [1] for more details on single-machine, parallel-machine and dedicated-machine scheduling problems with deteriorating jobs.

On the other hand, scheduling problems with group technology have been studied by Baker [2], Cheng et al. [3], Ham et al. [4], Mitrofanov [5], Ozden et al. [6], Webster and Baker [7], to name a few. To the best of our knowledge, only a few results concerning scheduling problems with deteriorating jobs and group technology simultaneously are known. Since longer setup or preparation might be necessary as food quality deteriorates or a patient's condition worsens, Wu et al. [8] considered a situation where the group setup times and job processing times are both described by a simple linear deterioration function. They showed that the makespan and the total completion time problems remain polynomially solvable under the proposed model. Wang et al. [9] considered the same model with Wu et al. [8], but with proportional deterioration. They proved that the makespan minimization problem and total weighted completion time minimization problem can be solved in polynomial time. Wang et al. [10] considered the same model with Wu et al. [8], but with a more general linear deterioration. For single machine group scheduling, they proved that the makespan minimization problem can be solved in polynomial time. Wang et al. [11] considered a single machine scheduling problem with deteriorating jobs, ready times and group technology, in

* Corresponding author.
E-mail address: wangjianjun_2008@163.com (J.-J. Wang).

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which the group setup times are assumed to be known and fixed. For a special case, they showed that the makespan minimization problem can be polynomially solvable. More recent papers which have considered scheduling jobs with deteriorating jobs and group technology include Yang [12], Yang and Yang [13], Wei and Wang [14], Cheng et al. [15], Bai et al. [16], Lee and Lu [17], Huang and Wang [18], Wang et al. [19], Xu et al. [20], Lu et al. [21], Wang and Wang [22], and Yin et al. [23].

However, traditional research on the scheduling problem with deteriorating jobs assume that all jobs to be processed have a single criterion. In real production settings and service environments, scheduling decisions are made with respect to bicriterion (multicriteria) performance rather than a single criterion (T'kindt and Billaut [24]). In this paper we consider a bicriterion scheduling problem with deteriorating setup times and deteriorating job processing times. This bicriterion model was proposed by Cheng et al. [3]. The remainder of this paper is organized as follows. In Section 2 we provide the notation and formulation of the problem. Section 3 deals with the makespan minimization problem. Concluding remarks are given in the last section.

2. Notation and problem statement

In this section, the notation that is used throughout the paper will be introduced first, followed by the formulation of the problem.

Notation.

G	the number of groups $(G \geqslant 2)$
G_i	group $i,\ i = 1, 2, \ldots, G$
n_i	the number of jobs in $G_i,\ i = 1, 2, \ldots, G$
n	the total number of jobs i.e., $(n_1 + n_2 + \ldots + n_G = n)$
J_{ij}	job j in $G_i,\ i = 1, 2, \ldots, G,\ j = 1, 2, \ldots, n_i$
δ_i	the deterioration rate of setup time for G_i
α_{ij}	the deterioration rate for the jth job in G_i
s_i	the actual setup time of G_i
p_{ij}	the actual processing time of J_{ij}
ω_{ij}	the relative importance for the jth job in G_i
C_{ij}	the completion time for the jth job in G_i
$C_{ij}(\pi)$	the completion time of J_{ij} under schedule π

There are n independent non-preemptive jobs to be scheduled for processing on a single machine. It is assumed that all jobs are available at time t_0, where $t_0 > 0$, without idle time and at most one job at a time. All jobs are classified into $G \leqslant n$ groups. Jobs in the same group have to be processed contiguously. A setup time precedes the processing of each group. The actual job-processing time of J_{ij} is a linear function of its starting time t, that is,

$$p_{ij} = \alpha_{ij} t, \quad i = 1, 2, \ldots, G, \quad j = 1, 2, \ldots, n_i.$$

Moreover, the actual setup time of G_i is also a linear function of its starting time t and as follow:

$$s_i = \delta_i t, \quad i = 1, 2, \ldots, G.$$

Given a schedule, the completion time C_{ij} for each job j in group i is easily determined. The quality of a schedule is measured by two criteria: the total weighted completion time $\sum\sum \omega_{ij} C_{ij} = \sum_{i=1}^{G} \sum_{j=1}^{n_i} \omega_{ij} C_{ij}$ and the maximum cost $f_{\max} = \max\{f_{ij}(C_{ij}) | i = 1, 2, \ldots, G,\ j = 1, 2, \ldots, n_i\}$, where all cost functions f_{ij} are nondecreasing in the job completion times. It is given that the primary criterion is the total weighted completion time and the secondary criteria is the maximum cost. Our primary criterion is one of the most important scheduling criteria. The objective is to minimize $f_{\max}$ on the set of schedules minimizing $\sum\sum \omega_{ij} C_{ij}$. Let GT denote the group scheduling problem. Using the conventional notation (Graham et al. [25]), the problem of minimizing $f_{\max}$ on the set of schedules minimizing $\sum\sum \omega_{ij} C_{ij}$ is denoted as $1|GT,\ s_i = \delta_i t,\ p_{ij} = \alpha_{ij} t|(\sum\sum \omega_{ij} C_{ij},\ f_{\max})$.

3. Transformation of problem and the polynomial time algorithm

3.1. Formulating the bicriterion problem as a single criterion problem

We introduce precedence constraints on a set of groups and on the set of jobs for each group and formulate our bicriterion problem as a single criterion problem to minimize $f_{\max}$ subject to these precedence constraints. First of all, we give two lemmas.

Lemma 1. *If the sequence of groups is fixed, then a schedule minimizes the total weighted completion time is obtained when* $\frac{\alpha_{ij}}{\omega_{ij}(1+\alpha_{ij})}$ *of each job in* G_i $(i = 1, 2, \ldots, G)$ *is increasing.*