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Avalanche duration time in a simple heterogeneous Olami–Feder–Christensen model

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ABSTRACT

A modified version of the Olami–Feder–Christensen model has been introduced to consider the difference between avalanche duration distribution. The duration time of our model well demonstrates the power-law behavior and finite size scaling. Relationship between earthquake duration time and average size has been discussed. It gives a power-law behavior and provides a new evidence of self-organized criticality. We have investigated foreshock and aftershock and discovered that the probability of middle-size earthquake in the aftershock was significantly greater than others. The real data of Sichuan earthquake is in alignment with our model.

1. Introduction

Self-organized criticality (SOC) was first introduced by Bak et al. in 1987 [1,2], which was proposed to reflect the statistical features of extended stationary state and dissipative dynamical systems. The criticality is characterized by a power-law distribution of the sizes of relaxation events without any fine tuning external parameters. SOC [3] has been empirically observed in diverse complex systems, e.g., fractal theory [4], chemical reactions [5], evolutionary population dynamics [6–12], forest burns [13], epidemics [14–16], heart attacks [17], market crashes [18], opinion formation [19], languages behavior [20], and more close of the topic in this article, earthquake [21–25].

To study the SOC behavior of earthquake, some key dynamical models have been proposed. A representative one is the Burridge–Knopoff (BK) spring model [26], in which an earthquake fault is simulated by an assembly of blocks mutually connected via elastic springs, with which the system is slowly driven by the external force. Another more phenomenological and extensively studied statistical model is the Olami–Feder–Christensen (OFC) model [21], which was firstly introduced by Olami et al. in 1992 as a simplification of the BK model. The OFC model is regarded as a typical nonconservative model of earthquakes, since it may produce well-known critical features such as the Gutenberg–Richter (GR) law [27] and Omori law [28]. Much literature [29–34] concerns the effect of generalized interactive dynamics on criticality behaviors of the OFC model, especially on fixed topological structure such as homogeneous lattice.

To reflect this, Baiesi and Paczuski proposed a metric to quantify correlations between earthquakes basing on scale-free networks, and they demonstrated that typical seismic events were strongly correlated with a few preceding ones [35]. Such a measure leads to an immediate classifications of foreshocks, mainshocks and aftershocks. Peixoto and Prado studied the

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epicenter network emerging from the OFC model [36], they formed direct networks and demonstrated the evident difference between conservative and nonconservative regimes. We found that these works assumed that energy delivery on each edge had a random or uniform, which could not reflect the real situation.

In reality, the transmission of seismic energy or force is usually inhomogeneous [37–42], thus it is reasonable that high-level heterogeneity in the earthquake systems is presented. In Refs. [42,43], we are inspired by the fact that tectonic plates possessing higher stress can be affected by adjacent plates much easier. In order to simulate this phenomenon, we introduce edge weight which determines how the energy is transferred from one point to another on the coupled-map lattice, to investigate the SOC behavior on the heterogeneous network.

In the study of earthquake, the duration time of seismic activity mainly consists of two types, namely duration time of earthquake and duration time of aftershock. The purpose of investigating the duration time of earthquake is to know the accurate stop time, which is the scientific basis for the earthquake prediction. If an earthquake time series mixed with the aftershocks, we would cause a high estimate parameter "a" in the GR law and wrong calculation of parameter "b", leading to the corresponding seismic risk overestimated. The study of aftershock duration time aims to discuss the aftershock itself and it can provide important basis for earthquake reconstruction, stability of social order and so on. Above all, it is necessary to discuss the duration time of earthquake in our model. This work aims to study the self-organized criticality behavior of the non-conservative modified OFC model, including earthquake duration time distribution, the foreshock, the aftershock behavior of earthquake and so on. In the remainder of this paper, we firstly specify our modified model of OFC, then, we present the main results, and at last, we will summarize the conclusions.

2. Model

In a typical OFC model, given a two-dimensional $L \times L$ lattice, each node is assigned a 'stress' variable F_i ($F_i \geqslant 0$), with a random initial value between $[0, 1]$. When all F_i are smaller than a threshold $F_{th} = 1$, they are increased at a constant rate. Otherwise, when the stress of any node i excesses the threshold, it 'topples' and releases a fraction of stress αF_i to each of the four nearest neighbors, after which F_i is reset to 0. And the force of all four nearest-neighbors are increased to $F_{nn} \to F_{nn} + \alpha F_i$, where '$nn$' denotes the set of nodes in the neighborhood of i. The toppling events proceed when the stress of i's neighbor j also excesses the threshold, i.e., $F_{j \in nn(i)} \geqslant F_{th}$. Such an avalanche behavior ceases when all nodes have a stress smaller than the threshold F_{th}. And, the duration of an avalanche, t_d, is the time duration of how long avalanche lasts.

It is well known that earthquakes occur as a result of the relative motion of tectonic plates and the seismic energy is released in the form of earthquake waves (primary wave or secondary wave). This process takes place from the epicenter, which is below the earth surface and spreads through the elastic vibration of the rocks. Due to different geological conditions, the earthquake wave in the rock will spread with different velocities and rates of decay. This will cause different energy decay in different geological conditions, therefore the heterogeneity of energy transfer occurs.

Heterogeneous OFC model. It is well known that the relative motion of tectonic plates plays a key role in the occurrence of earthquake, which leads to the accumulation of energy at the boundaries. When the accumulated stress cannot be neutralized by the friction, it has to be released intermittently. Since tectonic plates undertaking large stress are much more easily to be affected by adjacent plates, they possess an enhanced ability to collect seismic energy [43]. In this regard, we define the weight on each edge

$$w_{ij}(t) = \left[F_i(t) + F_j(t)\right]/2, \tag{1}$$

The parameter w_{ij} is used to adjust the rule of energy transmission. The parameter α is updated as:

$$\alpha \to \alpha_j(t) = a \times \frac{w_{ij}(t)}{\sum_{j \in nn(i)} w_{ij}(t)} = a \times \frac{F_i(t) + F_j(t)}{4F_i(t) + \sum_{j \in nn(i)} F_j(t)}, \tag{2}$$

where $\alpha_j(t)$ measures the level of local conservation, and a reflects the average level of nonconservation in the system [34,43]. Eq. (2) implies that the transmission of earthquake energy depends on the local seismic condition governed by stress distribution, so we introduce heterogeneity into the earthquake model.

3. Results

The heterogeneous OFC model considers a discrete system of blocks on a square lattice of size $L \times L$, interconnected by elastic springs and driven by a slowly moving plate. The total force acting on a given block, F_i, evolves according to the following rules: If all forces are smaller than a threshold value $F_{th} = 1$, all forces increase at the same constant rate, mimicking a uniform tectonic loading. At one time step a given site $F_i \geqslant F_{th}$, an avalanche begins (mentioned in detail above) and ends until no further site is unstable, then a new time step begins. The avalanche size s is defined as the total number of instable sites during an avalanche. There are two ways for defining an avalanche size. In the first case, each site can only topple once during an avalanche, thus it restricts the maximum size of an avalanche as the system size, and we label it as s_r. In the second case, each node can topple more than once during an avalanche, which relaxes the restriction of maximum avalanche size, and it is labeled as s_{nr}. Also, the duration of an avalanche, t_d, is how long the avalanche lasts. As you know, a local perturbation will spread to (some) nearest-neighbor sites ($t_d = 1$), then to next-nearest neighbors ($t_d = 2$), and under the domino