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Profile of the unique limit cycle in a class of general predator–prey systems

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ABSTRACT

Many predator–prey systems with oscillatory behavior possess a unique limit cycle which is globally asymptotically stable. For a class of general predator–prey system, we show that the solution orbit of the limit cycle exhibits the temporal pattern of a relaxation oscillator, when a certain parameter is small.

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1. Introduction

With the wide applications in the natural world, the predator–prey systems has been one of the important topics in ecology and mathematical biology. Along with the development of modern mathematics, the predator–prey systems have been by using qualitative analysis and stability theory. For the research of limit cycles, in 1975 Freedman and Waltman [4,5] used various techniques for establishing the existence of limit cycles. In 1981 Cheng [3] (see also Liou and Cheng [14]) published a result giving a criterion for the uniqueness of limit cycles for a special class of predator–prey models. In 1950's, Zhang proved a uniqueness theorem of limit cycles of generalized Liénard equations, which was later recorded in [25] in 1986. Zhang's result was used by Kuang and Freedman [13] to consider a Gause type predator–prey system:

$$\begin{cases} \dot{x} = xg(x) - \zeta(y)p(x), \\ \dot{y} = \eta(y)(-\gamma + q(x)). \end{cases}$$

in [13], they converted this predator–prey model to a Liénard equation, then showed that the new model satisfies the conditions in [25], consequently proved the uniqueness of limit cycle of this predator–prey system. Models of this type were introduced by Gause et al. [6], and since then, variations of this model have been utilized in Armstrong [1], Hassell [7], Hassell and May [10], and Rosenzweig [19], Alberecht et al. [2], May [16], Rosenzweig [18].

In 2009, Hsu and Shi [11] studied a predator–prey system in the particular form:

$$\begin{cases} \frac{du}{dt} = u(1 - u) - \frac{muv}{a+u}, \\ \frac{dv}{dt} = -dv + \frac{muv}{a+u}, \end{cases} \quad (1.1)$$

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where $a, m, d > 0$. The system (1.1) is often called Rosenzweig–MacArthur predator–prey system from the seminar work of Rosenzweig and MacArthur [20]. Hsu and Shi [11] considered the relaxation oscillator profile of the limit cycle of (1.1) by a careful phase portrait analysis and estimates. It is important to know whether such profile is special only to (1.1) or it holds for a more general class of predator–prey systems. Hence the objective of this paper is to study the dynamical properties of a general predator–prey systems, in particular, the asymptotic behavior of the limit cycle.

In this paper, we consider a class of more general predator–prey systems in the form

$$\begin{cases} \frac{du}{dt} = ug(u) - vp(u), \\ \frac{dv}{dt} = v(-d + p(u)), \\ u(0) \geq 0, \quad v(0) \geq 0, \end{cases} \quad (1.2)$$

where $d > 0$, the functions $g(u)$ and $p(u)$ are sufficiently smooth so that the existence, uniqueness, and continuous dependence on parameters of solutions to the initial-value problem are satisfied. The functions $u(t)$ and $v(t)$ represent the prey and predator populations, respectively, at a given time $t \geq 0$. In this paper, we assume that the functions $g(u)$ and $p(u)$ in (1.2) satisfy

(H1): $g \in C^2(\mathbf{R}^+)$, $g(0) > 0$, there exists $k > 0$, such that for any $u > 0$, $u \neq k$, $g(u)(u - k) < 0$ and $g(k) = 0$.

(H2): $p \in C^2(\mathbf{R}^+)$, $p(0) = 0$, $p'(u) > 0$ for any $u \geq 0$, and there exists $a \in (0, k)$ such that $p(a) = d$.

(H3): Define $F(u) = \frac{ug(u)}{p(u)}$ if $u > 0$ and $F(0) = \frac{g(0)}{p'(0)}$. Then $F \in C^2(\mathbf{R}^+)$. We assume there exists $a_* \in (0, k)$, such that for any $u > 0$, $u \neq a_*$, $F'(u)(u - a_*) < 0$ and $F'(a_*) = 0$.

It is known that (see Hsu [8]) if (H1)–(H3) are satisfied, then (1.2) possesses a unique coexistence equilibrium point $(a, F(a))$. The local stability of $(a, F(a))$ depends on the sign of $F'(a)$: when $a_* < a < k$, then $F'(a) < 0$ and $(a, F(a))$ is locally asymptotically stable; and when $0 < a < a_*$, then $F'(a) > 0$ and $(a, F(a))$ is unstable. Moreover the global stability of $(a, F(a))$ when $a_* < a < k$ can be established through a Lyapunov functional or Dulac criterion under some extra conditions (see [8,9]). On the other hand, when $0 < a < a_*$, the instability of $(a, F(a))$ implies the existence of a periodic orbit from the Poincaré–Bendixon theory. The uniqueness of the periodic orbit will make the periodic orbit a limit cycle—the attractor for the predator–prey system. Since the work of Cheng [3], the uniqueness of the limit cycle in (1.2) has been proved under some extra conditions [12,24]. Here we cite a result of Kuang and Freedman [13]: if (H1)–(H3) are satisfied, and also

(H4): for all $0 \leq u \leq k$, $u \neq a$, we have $\frac{d}{du} \left(\frac{p(u)F'(u)}{-d+p(u)} \right) \leq 0$,

then the limit cycle of (1.2) is unique and is global asymptotically orbital stable. Moreover, we can verify the uniqueness of limit cycle holds if (H1)–(H3) are satisfied, and also

(H4'): $F \in C^3(\mathbf{R}^+)$, and $uF'''(u) + 2F''(u) \leq 0$ for $0 \leq u \leq k$,

which can be obtained from results in [22,23].

We also recall that the growth rate of the prey is of logistic type if $g(u)$ is strictly decreasing, and it is of weak Allee effect type if $g(u)$ is increasing for $0 < u < c$ and is decreasing for $c < u < k$. Conditions (H1) and (H3) allow for either type of growth. For example, $g(u) = k - u$ is a logistic growth; for $g(u) = (k - u)(u + a)$, it is weak Allee effect type when $0 \leq a < k$, and it is logistic type when $a > k$. Some examples of $p(u)$ are Holling type II functional response $p(u) = mu/(b + u)$, or Ivlev type as $p(u) = m(1 - e^{-bu})$.

Our result here generalizes the one in [11], in which the relaxation oscillation profile of the limit cycle in a predator–prey model was first studied. An earlier work for relaxation oscillator in predator–prey model appeared in [15]. For many other mathematical models with limit cycle behavior and small parameters, such relaxation oscillation have been well-documented in, for example, [17,21]. For such relaxation oscillation profile, the prey population $u(t)$ is near zero for a very long period when d is small (see Figs. 2 and 3 for illustration). Biologically this means the prey population is vulnerable to extinction even with small stochastic perturbations.

We prove our main results in Section 2 for the case $d \rightarrow 0$. We will use δ_i and C_i , ($i \in \mathbf{N}$), to denote various positive constants. These constants are independent of d in Section 2. We give an example and some numerical simulations to illustrate our results in Section 3.

2. Asymptotic behavior of the limit cycle for d small

In this section, we consider the asymptotical profile of the limit cycle of (1.2). We assume that $0 < a < a_*$, and the conditions (H1)–(H4) (or (H1)–(H4')) hold. We define

$$f(u, v) = ug(u) - vp(u), \quad g(u, v) = v(-d + p(u)). \quad (2.1)$$

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