Applied Mathematics and Computation 242 (2014) 518–527

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Flow of granular materials with slip boundary condition: A continuum–kinetic theory approach

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ARTICLE INFO

Keywords:
Slip boundary condition
Granular materials
Continuum mechanics
Kinetic theory
Shear flow

ABSTRACT

We study the steady fully developed flow of granular materials between two horizontal flat plates, subject to slip at the walls. The constitutive model for the stress tensor is based on ideas in continuum mechanics and kinetic theory. The constitutive equation used in our study is a model proposed by Rajagopal et al. (1994) [24], and the material properties such as viscosity and the normal stress coefficients are derived using the kinetic theory approximation proposed by Boyle and Massoudi (1990) [2] which includes the effect of the gradient of volume fraction. The slip boundary condition is based on the particle dynamics simulation results of Rosato and Kim (1994) [30]. The governing equations are non-dimensionalized, and the resulting system of non-linear differential equations is solved numerically. The results for the velocity profiles and the volume fraction profiles are presented.

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1. Introduction

The earliest study of granular materials can be dated perhaps to the Middle Ages, where the hourglass or the sand clock were used by scholars to regulate their studies and by the clergy to time their sermons [31]. After industrialization, the basic issues were the need of predicting the stresses in container walls to avoid failure, minimizing the erosion in solids transport, and promoting the mixing in fluidized bed, etc. Furthermore, many natural phenomena require an understanding of flow of bulk solids, such as debris flow [7], and snow or avalanches [32]. Other examples of bulk solids include coal, sand, ore, grains, cereals, and so on.

Granular materials can be treated as the limiting case of two-phase flow at high solid concentration and high solid-to-fluid density ratios. For two-phase flows, the interstitial fluid interacts with the solid component, but for bulk solids, the solid components dominate, especially when the interstitial fluid is a gas, the influence of fluid component on the solid component is negligible. A powder is composed of particles up to 100 μm (diameter) with further sub-division into ultrafine (0.1–1 μm), superfine (1–10 μm), or granular (10–100 μm) particles. A granular solid consists of materials ranging from about 100 to 3000 μm [3].

It is well known that granular materials can sustain shear stress without deformation, and the critical shear stress at which the shearing begins to occur depends on the normal stress. It is also widely observed that granular materials can exhibit normal stress differences under shearing motion, related to the phenomenon known as dilatancy. Also it is known that (densely) packed granular materials exhibit yield stress. A simple physical interpretation could be that, for tightly packed

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http://dx.doi.org/10.1016/j.amc.2014.05.093
0096-3003/Published by Elsevier Inc.

granular materials, the granules must expand normal to the direction of flow in order to increase the void volume. This phenomenon is very similar to a non-linear slab undergoing dimensional changes in the direction perpendicular to the plane of shearing [22], and also there is a phenomenon known as dilatancy, first observed by Reynolds [27].[1] Also it is known that (densely) packed granular materials can exhibit yield stresses. Non-equal normal stress differences are also depicted in die swell, rod-climbing effect, or secondary flows in plate/cylinder system [35].

In general, to understand the mechanics of granular materials one can either perform simple experiments to characterize their behavior or formulate a theory where the response of granular materials can be studied. Review articles by Savage [31], Hutter and Rajagopal [13] and books by Mehta [21], Duran [8], Antony et al. [1] and Rao and Nott [25] address many of the interesting issues in the field of granular materials. In general, there are at least two types of methods to study granular materials, namely, the continuum approach and the statistical approach. In the continuum approach, we treat the material properties as a continuous function of space and time, and assume that the medium can be divided indefinitely without losing its properties. Rajagopal et al. [24] proposed a mathematical model for granular-like material, in which the Cauchy stress tensor depends on the symmetric part of velocity gradient and the density gradient. Massoudi and Mehrabadi [18] showed that, this granular model is able to predict the dilatancy effect which is related to the normal stress effects. At the same time, if a proper representation is given to some of the material parameters, this model would also comply with the Mohr–Coulomb criterion.

Among the statistical approaches, particle dynamics approach [11] and modified kinetic theory [14] for dense approach are two widely used methods to study granular flow. The kinetic theory approach is based on a statistical velocity distribution function, where the fluctuation energy of particle, also referred to as the "granular temperature", is used to describe the random motion of flowing granules. The magnitude of the granular temperature is proportional to the mean square of the random particle velocity. For low-speed flow where the collisions between particles are rare, the basic assumptions of kinetic theory are not valid any more [34]. There are many models based on kinetic theory. (See [4,9]).

In many applications, a no-slip boundary condition is used, but for granular materials, in general, slip may occur at the walls. For example, Johnson and Jackson [15] formulated the constitutive relations and boundary conditions for a cohesionless granular material by using the conservation of momentum and the fluctuation energy, and incorporating friction into a boundary at which some particles collide and the rest of particles slide. The momentum and energy transfer of the sliding particles was depicted by Coulomb friction, and that due to the colliding particles was characterized by a coefficient of specularity. Richman [28] derived the expression for slip velocity based on a modified Maxwellian velocity distribution for flowing particles, by modeling the granules as identical, smooth and nearly elastic spheres. Savage and Dai [33] and Rosato and Kim [30] used particle dynamic approach to study the interaction between flowing particles and bumpy walls.[2]

Boyle and Massoudi [2] derived a continuum–kinetic theory constitutive equation by utilizing the ideas of Enskog's dense gas theory; their model can predict normal stress differences arising due to density gradient. In this paper, we will use this model to study the flow of granular materials between two shearing plates, subject to slip at the walls, where the slip boundary condition is based on the simulation results from Rosato and Kim [30].

2. Governing equations

In the absence of thermo-chemical or electro-magnetic effects, the governing equations based on a continuum–kinetic theory approach (see [17]) are given by the conservation of mass, linear momentum equation, angular momentum equation, and the pseudo-energy equation.

The equation for conservation of mass is written as

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{v}) = 0, \tag{1}$$

where $\partial/\partial t$ is the partial derivative with respect to time, ρ is the density, $\operatorname{div}(\cdot)$ is the divergence operator, and $\boldsymbol{v}$ is the velocity vector. The linear momentum equation is written as

$$\rho \frac{D\boldsymbol{v}}{Dt} = \operatorname{div}\boldsymbol{T} + \rho \boldsymbol{b}, \tag{2}$$

where D/Dt is substantial time derivative

$$\frac{D(\cdot)}{Dt} = \frac{\partial(\cdot)}{\partial t} + [\operatorname{grad}(\cdot)]\boldsymbol{v} \tag{3}$$

[1] Many of the existing theories for the flowing granular materials use this observation to relate the applied stress to the voidage and the velocity. Reiner [26] proposed and derived a constitutive relation for wet sand whereby the concept of dilatancy is given a mathematical structure. Massoudi [20] generalized Reiner's model to include the effects of volume fraction.

[2] The effect of boundaries on the flow of granular materials has been studied by many researchers (see for example, [12,10,6,5]). Whether one uses the continuum approach or the kinetic theory approach, slip may occur at the walls, especially when the interstitial fluid is a gas, and therefore the classical assumption of adherence boundary condition at the wall no longer applies (see [16] for a review of slip flow).