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Heat and mass transfer of two-layer flows of third-grade nano-fluids in a vertical channel

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ABSTRACT

The heat and mass transfer of two-layer flows of non-Newtonian (third-grade) fluid in a vertical channel is investigated in details, when one layer of water is nano-fluid and the other is clear, with the viscous dissipation. In each layer, the boundary-layer flow is governed by coupled nonlinear ordinary differential equations (ODEs). The systems of coupled nonlinear ODEs are solved analytically by means of a Mathematica package BVPh 2.0 based on the homotopy analysis method (HAM), an analytic approximation method for highly nonlinear problems. The influence of the physical parameters is studied in detailed. It is found that the layer of nano-fluid has many different properties from that of clear fluid. This paper also illustrates the validity and power of the HAM-based Mathematica package BVPh 2.0 for some complicated boundary-layer flows.

1. Introduction

The mixed or combined convection arises in many situations such as, natural convection in the presence of ambient fluid circulations, the externally induced flow in heated channels, etc. The flow and heat transfer in mixed convection situations are topic of great interest due to its applications in chemical processing equipment, cooling of electronic circuitry by means of fan, solar technology, nuclear reactors cooled during emergency shut down, etc. Sparrow et al. [1] was the first to study the mixed convection flows, followed by Sparrow and Gregg [2], Tao [3], Szewczyk [4] and Merkin [5]. Due to its wide applications, a lots of investigations were made on fully developed mixed convection flows in a vertical channel. Rajagopal and Na [6] studied third-grade fluid between two infinite parallel vertical plates. Aung and Worku [7] investigated flows in a vertical channel with asymmetric wall temperature. Further, Aung and Worku [8] examined mixed convection flows in a vertical channel with different wall temperatures. They found that flow is reversed for large values of buoyancy parameter. Massoudi and Christie [9] studied the influences of the non-Newtonian nature of fluid on the skin friction and heat transfer. Barletta [10] investigated combined forced and free convection flows in a rectangular duct for laminar, fully developed regime. Boulama and Galanis [11] presented analytical solutions for fully developed mixed convection between parallel plates. Recently, Baoku and Olajuwon [12] investigated viscoelastic third-grade fluid past an infinite vertical insulated plate subject to suction across the boundary layer. Some more recent studies for flows of third-grade fluid can be consulted through the

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Refs. [13–18]. Note that the aforementioned studies investigate models with only a single-fluid. However, in practice, most of the problems relating to petroleum industry, geophysics, plasma physics, magneto-fluid dynamics, etc., involve multi-fluid flow situations. A number of complex interfacing transport phenomena may take place in non-isothermal multi-fluid systems. An important assumption usually met in these models is the interfacial thermal and chemical equilibrium between the fluids. Beckermann et al. [19] performed numerical investigation to analyze the flow and heat transfer between a fluid layer and a porous layer inside a rectangular enclosure. Kimura et al. [20] studied the influence of ratio of the depth of two-layers on the heat transfer. Malashetty et al. [21] presented analytical solutions for two region vertical enclosure with one region electrically conducting and the other electrically non-conducting. Kumar et al. [22] investigated the influences of the governing parameters on the flow and heat transfer for two-layer fluid with one region filled with micropolar fluid and the other with clear fluid. Umavathi et al. [23] examined three-layer flows with a porous media sandwiched between clear fluids. Nikodijevic et al. [24] obtained closed-form solutions for magnetohydrodynamic Couette flow of two immiscible fluids. Parthab et al. [25] studied two-layer mixed convective flow and heat transfer in a vertical channel with one region filled with conducting fluid and other with nonconducting ones.

Nanotechnology grows rapidly due to its immense applications in many electronic devices, vehicles, space craft, metrology, artificial organs and cooling applications of nano-fluids, and so on. Choi [26] is the first to use the term nano-fluid, who proposed that nanometer sized metallic particles can be suspended in industrial heat transfer fluids. Therefore, a nano-fluid is a suspension of nano-particles (metallic, non-metallic, or polymeric) in a conventional base fluid which enhances its heat transfer characteristics. Enhanced thermal properties of nano-fluids enable them to use in automotive industry, power plants, cooling systems, computers, etc. An in-depth review on nano-fluids can be found in the book by Das et al. [27] and in the review paper by Wang and Mujamdar [28]. Recently, the flows due to a mixed convection in a vertical channel in nano-fluids have drawn considerable attention. Kuznetsov and Nield [29] investigated natural convective boundary layer flows of a nano-fluid past a vertical plate. Xu and Pop [30] studied the influences of nano-particle volume fraction on the temperature and velocity distributions for fully-developed mixed convection flow in a vertical channel filled with nano-fluids. Gorsan and Pop [31] investigated mixed convection nano-fluid flow in a vertical channel for different physical parameters. Xu et al. [32] obtained series solutions for mixed convection flow of a nano-fluid in a vertical channel. Recently, Gorder et al. [33] investigated fully-developed two-layer fluid flows in a vertical channel with one region filled with nano-fluid and the other filled with clear fluid. In the same two-layer model by Gorder et al. [33], Farooq and Lin [34] proposed new non-dimensional quantities for physical parameters and exhibited the flow reversal phenomenon for sufficiently high buoyancy. Vajravelu et al. [35] investigated heat and mass transfer properties of three-layer fluid flow in which nano-fluid layer is squeezed between two clear viscous fluid.

To the best of our knowledge, the two-layer model with one region filled with non-Newtonian third-grade clear fluid and the other with non-Newtonian third-grade nano-fluid has never been investigated. Due to the non-Newtonian third-grade fluid, the flows are governed by more complicated, coupled ODEs in each layer. The systems of these coupled non-linear ODEs are solved by means of the Mathematica package BVPh 2.0 [40], which is valid for nonlinear boundary-value or eigen-value problems with boundary conditions at multiple points, governed by coupled nonlinear ODEs. The BVPh 2.0 is free available online (http://numericaltank.sjtu.edu.cn/BVPh.htm). It is based on the homotopy analysis method (HAM) [36–39], an analytic approximation method for highly nonlinear problems. Unlike perturbation techniques, the HAM has nothing to do with small/large physical parameters, so that it is essentially a non-perturbation method. Besides, based on the homotopy in topology, the HAM provides us great freedom to choose the equation-type and base function of equations for high-order approximations. Especially, unlike all other analytic techniques, the HAM provides us a convenient way to control and adjust the convergence of solution series, so that the convergence of solution series can be guaranteed. Therefore, unlike other traditional analytic methods, the HAM is valid for highly nonlinear problems. The HAM has been widely applied to solve lots of nonlinear problems in science, engineering and finance. The BVPh 2.0 [40] is a HAM-based Mathematica package, which provides us an easy-to-use tool for nonlinear boundary-value or eigenvalue problems governed by coupled nonlinear ODEs.

In this paper, the complicated system of the coupled nonlinear ODEs in each layer of non-Newtonian fluid is solved conveniently by means of the BVPh 2.0. The influence of physical parameters on the flow, temperature, concentration, heat and mass transfer is investigated in detail. Reversed flow is observed for sufficiently large values of mixed convection parameter. These are helpful to deepen and enrich our understandings about the considered two-layer flows of third-grade fluid in a vertical channel.

2. Problem formulation

Consider steady-state, laminar, incompressible flow between two infinite vertical parallel plates extending in the x and z directions, with x-axis upwards and y-axis pointing to the right, as shown in Fig. 1. The region-I in the domain $0 \leqslant y \leqslant h$ is filled with a third-grade clear-fluid, and the region-II in the domain $-h \leqslant y \leqslant 0$ is occupied by a third-grade nano-fluid, respectively. In each layer, let ρ_i denote the density, μ_i the viscosity, α_i the thermal diffusivity and γ_i the non-Newtonian parameter, respectively, where $i = 1, 2$. The right and left walls have temperatures T_{w1} and T_{w2}, respectively. In the two regions, the fluid thermo-physical properties keep the same.