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# Numerical and analytical solutions for Falkner–Skan flow of MHD Maxwell fluid

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ABSTRACT

The present research examines the Falkne–Skan flow of magnetohydrodynamic (MHD) Maxwell fluid. Both analytic and numerical solutions are established for the governing problem. The variations of viscoelastic and magnetic parameters are emphasized.



## 1. Introduction

Investigation of boundary layer flows has attracted the attention of various researcher in view of their applications in engineering processes. In particular the Falkner–Skan equation under different aspects has been extensively studied by the researchers in boundary layer theory [1]. Although the solution of the governing nonlinear problem in viscous fluid have been presented numerically by Hartree [2], Howarth [3], Asaithambi [4], Cebeci and Keller [5], Sher and Yakhot [6] and Liu and Chang [7], but analytic solutions to such flow are still important. Therefore, Alizadeh et al. [8] studied the Falkner–Skan flow of hydrodynamic viscous fluid through the implementation of Adomian decomposition method (ADM). In continuation, the MHD Falkner–Skan flow of viscous fluid has been analyzed by Abbasbandy and Hayat [9]. In this paper the solution is given by using homotopy analysis method (HAM). Yao [10] also used the same method for the solution to Falkner–Skan wedge flow with the permeable wall of uniform suction.

It is well known that majority of the fluids in industry and engineering applications are non-Newtonian. Those fluids are generally categorized into the rate, differential and integral types. Much attention in the literature is devoted to the subclasses of differential type fluids. There is a simplest subclass of rate type fluids known as Maxwell fluid. This fluid model can describe the relaxation time effects in polymeric materials with low molecular weight. Some contributions have already made using Maxwell fluid. For example, Zhao et al. [11] analyzed diffusive convection in a Maxwell fluid saturated layers. Start up flow of fractional Maxwell fluid in a pipe was examined by Yang and Zhu [12]. Effects of thermal radiation for flow of Maxwell fluid in a channel with porous medium was studied by Hayat et al. [13] Abbas et al. [14] analyzed mixed convection feature in the stagnation point flow of Maxwell fluid. Jamil and Fatecau [15] developed Helical flows of Maxwell fluid between coaxial cylinders. Energetic balance for Stokes' first problem of Maxwell fluid was introduced by Zierep and Fetecau [16]. Tan and Masuoka [17] presented stability analysis for flow of Maxwell fluid in a porous space. Hayat et al. [18] discussed the hydrodynamic boundary layer flow of Maxwell fluid in the presence of mixed convection. Effects of slip condition on transient flow of Maxwell fluid was illustrated by Hayat et al. [19] MHD stagnation point flow of an upper convected

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Maxwell fluid over a stretching surface is addressed by Hayat et al. [20] Awais et al. [21] constructed time-dependent three dimensional hydrodynamic flow of Maxwell fluid. Influences of heat transfer and porous medium the flow of Maxwell fluid over an unsteady stretching surface are explained by Mukhopadhyay et al. [22] Shateyi [23] numerically discussed the flow of Maxwell fluid in the presence of thermophoresis and chemical reaction.

The objective here is to address the MHD effects in the Falkner–Skan flow of Maxwell fluid. Series and numerical solutions are constructed for the governing nonlinear problem. Series solution is obtained using the methodology in the Refs. [24–31]. Numerical solution is computed by Runge–Kutta method [32]. Optimal HAM is also utilized in the studies [33–40]. The effects of important parameters of interest on the flow quantities are also discussed. It should be noted that in optimal HAM the convergence control parameter can often be selected to minimize error.

## 2. Development of problem

We consider two-dimensional boundary layer flow of Maxwell fluid with stream-wise pressure gradient. A magnetic field of constant strength $B_0$ acts in the $y$-direction. The electric and induced magnetic fields are absent. The governing boundary layer equations can be written as [13]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right]=U\frac{\mathrm{d}U}{\mathrm{d}x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\left(u-U+\lambda_1 v\frac{\partial u}{\partial y}\right), \tag{2}$$

with the relevant conditions

$$u=0,\quad v=0 \text{ at } y=0, \tag{3}$$

$$u=U(x),\ \text{as } y\to\infty,$$

$$U(x)=ax, \tag{4}$$

where $u$ and $v$ are the velocity components, $U$ the inherent characteristic velocity, $\nu$ the kinematic viscosity, $\sigma$ the electrical conductivity, $\lambda_1$ the relaxation time and $\rho$ the fluid density.

Defining

$$\tau=\sqrt{\frac{U}{\nu x}}y,\quad \psi=\sqrt{\nu x U}f(\tau), \tag{5}$$

$$u=Uf'(\tau),\quad v=-\sqrt{\frac{\nu U}{x}}f, \tag{6}$$

the continuity equation is identically satisfied and Eqs. (1) and (2) give

$$\frac{\mathrm{d}^3 f}{\mathrm{d}\tau^3}+(1+\lambda M^2)f\frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2}+\left(1-\left(\frac{\mathrm{d}f}{\mathrm{d}\tau}\right)^2\right)-M^2\left(\frac{\mathrm{d}f}{\mathrm{d}\tau}-1\right)+2\lambda f\frac{\mathrm{d}f}{\mathrm{d}\tau}\frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2}-\lambda f^2\frac{\mathrm{d}^3 f}{\mathrm{d}\tau^3}=0, \tag{7}$$

$$f(0)=0,\quad f'(0)=0,\quad f'(+\infty)=1, \tag{8}$$

in which $\lambda=a\lambda_1$ and $M^2=\sigma B_0^2/\rho a$.

## 3. Series solution

In view of Eqs. (7) and (8), we write the following expression

$$f(\tau)=\sum_{q,n=0}^{+\infty}c_{q,n}\tau^q e^{-n\gamma\tau}, \tag{9}$$

in which the $c_{q,n}$ $(q,n=0,1,\ldots)$ are the coefficients to be determined and $\gamma>0$ is a spatial-scale parameter. Initial approximation and auxiliary linear operator chosen in the forms

$$f_0(\tau)=\tau-\frac{1-\exp(-\gamma\tau)}{\gamma}, \tag{10}$$

$$\mathcal{L}[\phi(\tau,\gamma;p)]=\left(\frac{\partial^3}{\partial\tau^3}+\gamma\frac{\partial^2}{\partial\tau^2}\right)\phi(\tau,\gamma;p). \tag{11}$$