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Structural stability for a linear system of thermoelastic plate equations

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ABSTRACT

The continuous dependence on parameters of strong solutions of initial boundary value problems for thermoelastic plate equation is established.

1. Introduction

We study the problem of continuous dependence on parameters of strong solution of the initial boundary value problem for the linear thermoelastic plate equation of the form:

$$u_{tt} - h\Delta u_{tt} + \Delta^2 u + \alpha\Delta\theta = 0, \quad x \in \Omega, \quad t > 0, \tag{1.1}$$

$$\theta_t - \mu\Delta\theta - \alpha\Delta u_t = 0, \quad x \in \Omega, \quad t > 0, \tag{1.2}$$

$$u = \frac{\partial u}{\partial \nu} = \theta = 0, \quad x \in \partial\Omega, \quad t > 0, \tag{1.3}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad x \in \Omega. \tag{1.4}$$

Here Ω is a bounded domain in R^n with sufficiently smooth boundary $\partial\Omega$; h, α, μ are given positive parameters; $u_0(x), u_1(x)$ and $\theta_0(x)$ are given initial functions, $u(x,t)$ and $\theta(x,t)$ are unknown functions that represent the vertical deflection and the temperature of the plate, respectively.

Exponential decay of energy integral of the system (1.1)–(1.4) for $h = 0$ with the homogeneous Dirichlet boundary conditions was studied by Kim [5] by using energy method. Rivera and Shibata [9] proved the existence of strong solutions of the problem (1.1)–(1.4), and showed the exponential decay of energy.

The first purpose of the present work is to prove a continuous dependence of the solution to the problem (1.1)–(1.4) on the coefficient h.

Liu and Zheng [8] proved the exponential stability of the thermoelastic plate model,

$$u_{tt} - h\Delta u_{tt} + \Delta^2 u + \alpha\Delta\theta = 0, \quad x \in \Omega, \quad t > 0, \tag{1.5}$$

$$\theta_t - \eta\Delta\theta + \sigma\theta - \alpha\Delta u_t = 0, \quad x \in \Omega, \quad t > 0, \tag{1.6}$$

with boundary conditions,

$$u = \frac{\partial u}{\partial \nu} = 0, \quad x \in \Gamma_0, \quad t > 0,$$

$$u = \Delta u + (1-\mu)B_1 u + \alpha\theta = 0, \quad x \in \Gamma_1, \quad t > 0.$$

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Here $\Gamma = \Gamma_0 \cup \Gamma_1$ with $\bar{\Gamma_0} \cap \bar{\Gamma_1} \neq \emptyset$. Then in 1998 [2], by using multiplier method it was shown that the energy of the system (1.5) and (1.6) under the boundary conditions:

$$u = (1-\kappa)\frac{\partial u}{\partial v} = 0,$$
$$\frac{\partial \theta}{\partial v} + \lambda\theta = 0,$$
$$\kappa(\Delta u + (1-\mu)B_1 u + \alpha\theta) = 0,$$

decays exponentially. Here κ is either 0 or 1.

On the other hand there has been many papers in the literature answering the question of structural stability of different equations: Structural stability for the model of non-isothermal flow in porous media which includes Brinkman effects was shown in [4], the continuous dependence of the solution on the Soret coefficient for Darcy model was proved in [7], the continuous dependence on Soret coefficients of solution of the double diffusive convective Brinkman equation with homogen boundary conditions was established in [3], structural stability for multidimensional FitzHugh–Nagumo equations on the diffusivity coefficient was shown in [1]. Many results of this character can be found in [10].

The second purpose of this paper is to prove the continuous dependence of the solution on the parameter β to the system (1.1) and (1.2) under the boundary conditions:

$$u = \frac{\partial u}{\partial v} = 0, \quad x \in \partial\Omega, \quad t > 0,$$
$$\frac{\partial \theta}{\partial v} + \beta\theta = F(x,t), \quad x \in \partial\Omega, \quad t > 0.$$

Throughout the paper, $||.||$ and $(,)$ denote the norm and inner product of $L^2(\Omega)$.

2. Continuous dependence on coefficient *h*

In this section firstly we obtain a priori estimate for solution of the problem (1.1)–(1.4) which allow us to prove the continuous dependence on h

Theorem 2.1. *If $[u,\theta]$ is a strong solution of the problem (1.1)–(1.4) then the following inequality holds*

$$||u_{ttt}||^2, \quad ||\nabla u_{ttt}||^2, \quad ||\Delta u_{tt}||^2, \quad ||\theta_{tt}||^2 \leqslant M_1(h).$$

Proof. First we differentiate the (1.1) and (1.2) two times with respect to t and then multiply the resulting equations in $L^2(\Omega)$ by $u_{ttt}, \ \theta_{tt}$ respectively so adding the obtained relations we get:

$$\frac{d}{dt}E_1(t) = -2\mu||\nabla\theta_{tt}||^2,$$

where

$$E_1(t) = ||u_{ttt}||^2 + h||\nabla u_{ttt}||^2 + ||\Delta u_{tt}||^2 + ||\theta_{tt}||^2.$$

So we get

$$E_1(t) \leqslant E_1(0)$$

and thus

$$||u_{ttt}||^2, \quad ||\nabla u_{ttt}||^2, \quad ||\Delta u_{tt}||^2, \quad ||\theta_{tt}||^2 \leqslant M_1(h). \tag{2.1}$$

where $M_1(h) = E_1(0)$. □

The following theorem is proved for the continuous dependence on coefficient h.

Theorem 2.2. *If $[u_1,\theta_1]$ and $[u_2,\theta_2]$ are strong solutions of the problem (1.1)–(1.4) then the following inequality holds*

$$||\omega_t||^2 + h_1||\nabla\omega_t||^2 + ||\Delta\omega||^2 + ||z||^2 \leqslant h^2 R(t). \tag{2.2}$$

Proof. To investigate continuous dependence on h, we let $[u_1,\theta_1]$ and $[u_2,\theta_2]$ be solutions to (1.1) and (1.2) for the same initial and boundary data but for different coefficients h_1 and h_2. Let us define the difference variables as

$$\omega = u_1 - u_2, \tag{2.3}$$
$$z = \theta_1 - \theta_2 \tag{2.4}$$