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Condition number and backward errors of nonsymmetric algebraic Riccati equation ☆

Lan-dong Liu [a,*], Shu-fang Xu [b]

[a] Department of Mathematics, China University of Mining and Technology, Beijing 100083, China
[b] School of Mathematics Sciences, Peking University, Beijing 100871, China

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ABSTRACT

The normwise condition number and two kinds of backward errors are considered when nonsymmetric algebraic Riccati equation (NARE) has the minimal nonnegative solution. Based on the techniques for the symmetric case, we apply the condition number theory developed by Rice to define condition number for NARE. The explicit expression is derived in a uniform manner. Meanwhile, two kinds of backward errors are defined and evaluated by the explicit formulas. Numerical experiments are listed to illustrate and compare the practical performance.

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1. Introduction

We consider nonsymmetric algebraic Riccati equation (NARE)

$$XCX - XD - AX + B = 0, \tag{1.1}$$

where A, B, C, D are real matrices of sizes $m \times m,\ m \times n,\ n \times m,\ n \times n$, respectively, and solution $X \in \mathbb{R}^{m\times n}$. NARE (1.1) has many applications in transport theory and Markov models [4,13]. The solution of practical interest is the minimal nonnegative solution. So we assume that

$$K = \begin{bmatrix} D & -C \\ -B & A \end{bmatrix} \tag{1.2}$$

is a nonsingular M-matrix to guarantee NARE (1.1) has the minimal nonnegative solution. The numerical algorithm of NARE (1.1) has attracted some authors' attention. Theory and numerical methods of nonsymmetric algebraic Riccati equation are well developed [3,1,2,4–6,8,10,11,26]. In particular, structure-preserving doubling algorithm [11,9,14,16] is a very efficient algorithm. For these algorithms details, please refer to the references therein.

Our interest here is to discuss the perturbation analysis of NARE (1.1). As is known perturbation analysis contains forward perturbation analysis and backward perturbation analysis. The purpose of forward perturbation is to ascertain the stability of an equation or a problem itself. The result of the forward perturbation analysis may be a perturbation bound, or a condition number, or a perturbation expansion. The back perturbation analysis is to test the stability of a computation or an algorithm,

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* Corresponding author.
E-mail address: Liuld@cumtb.edu.cn (L.-d. Liu).

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and ascertain the accuracy of an approximate solution. The result of backward perturbation analysis may be a backward error or a residual. For more details about perturbation theory, please refer to Higham [12] and Sun [19].

For algebraic Riccati equation some perturbation analysis results have been achieved, such as symmetric algebraic Riccati equation [26,29], discrete-time Riccati equation [20–22,15,23,24]. The study of perturbation analysis of NARE (1.1) was developed by Xu [26]. He presented the perturbation bound and the residual bound of NARE (1.1). Guo and Bai [10] discussed the perturbation bound and structured condition number about the minimal nonnegative solution of NARE (1.1). Recently some authors researched on the mixed and componentwise condition numbers of algebraic Riccati equation. For example, Zhou et al. [29] discussed symmetric algebraic Riccati equation, and Liu [17] discussed nonsymmetric algebraic Riccati equation. Xue et al. [28,27] considered the relative perturbation theory and its entrywise relatively accurate numerical solutions of M-matrix algebraic Riccati equations and M-matrix Sylvester equations.

The purpose of this paper is to evaluate the condition number and backward error of a minimal nonnegative solution to NARE (1.1). Condition number of NARE (1.1), as the measure of the sensitivity for the minimal nonnegative solution to small changes in the coefficient matrices, plays a key role in the perturbation theory. We will apply the condition number theory developed by Rice [18] to define condition number of NARE (1.1), and derive an explicit expression of condition number in a uniform manner.

Backward error is an important concept proposed by Wilkinson [25] in the 1960s. Now it becomes one of the basic tools to evaluate the quality of the calculated solution. This paper will discuss two kinds of backward errors of NARE (1.1) and derive their corresponding explicit expressions. This research is greatly influenced by Sun [23].

Throughout this paper we will use the following notations:

$\|\bullet\|_F$	Frobenius norm
$\|\bullet\|$	the operator norm induced by the Frobenius norm in an associated matrix space
$\|\bullet\|_2$	Euclidean vector norm
vec	vectorization operator, which stacks the columns of a matrix one under another
$\otimes$	Kronecker product
$\circ$	Hadamard product
I	identity matrix with the size determined by the context
Ω	$\mathbb{R}^{m\times m}\times\mathbb{R}^{m\times n}\times\mathbb{R}^{n\times m}\times\mathbb{R}^{n\times n}$
$\sigma(A,B,C,D)$	$\sqrt{\|A\|_F^2+\|B\|_F^2+\|C\|_F^2+\|D\|_F^2}$

This paper is organized as follows. In Section 2 some preliminary results are given. In Section 3 we define the condition number for the minimal nonnegative solution of NARE (1.1) and obtain a calculable explicit expression. In Section 4 two kinds of backward errors are defined, meanwhile the computable explicit expressions are given. In Section 5 the results are illustrated by some numerical examples.

2. Preliminaries

In this section we give some preliminary results. Lemma 1 [7] and Lemma 2 [26] are introduced.

Lemma 1. *Let K be a nonsingular M-matrix, then NARE (1.1) has a minimal nonnegative solution X, such that*

$$D_C = D - CX \tag{2.1}$$

and

$$A_C = A - XC \tag{2.2}$$

are all nonsingular M-matrices.

Lemma 2. *Let the map $f:\mathbb{R}^n\to\mathbb{R}^m$ be resolved into*

$$f(x)=\varphi(x)+g(x),$$

where $x\in\mathbb{R}^n$. Assume that

(1) $f(x),\ \varphi(x),\ g(x)$ *are continuous,*
(2) $\varphi(\alpha x)=\alpha\varphi(x),\ \alpha\in\mathbb{R}^+,\ \forall x\in\mathbb{R}^n$,
(3) $\lim_{\|x\|\to 0}\frac{\|g(x)\|}{\|x\|}=0.$

Then for an arbitrary given vector $p=(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n)^T$, here $\varepsilon_i>0\ (i=1,2,\ldots,n)$, it holds that

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