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Output regulation for heterogeneous linear multi-agent systems based on distributed internal model compensator

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ABSTRACT

This article considers robust output regulation of uncertain heterogeneous multi-agent systems in the case that all the agents have non-identical nominal dynamics. The directed communication graph contains a spanning tree and the exosystem is as its root. Since not all the agents can access the information of the exosystem, the distributed compensator is used for the unaccessible part. The dynamic state feedback control law and dynamic output feedback control law are proposed under this topological structure. Then we give a novel compact form and a general global method to solve the robust output regulation problem based on internal model principle. Finally, some examples are presented to illustrate the effectiveness of our results.

1. Introduction

In recent years, there has been extensive interest in the study of network systems. A networked multi-agent system is a dynamical system composed of a group of subsystems. Many pioneering contributions involved with distributed strategies that achieve consensus have been witnessed [1–3]. In multi-agent systems, the agents communicate information with each others to perform an identical task. Such control systems are often referred to as consensus control systems. The consensus problem requires an agreement to be reached that depends on the states of all agents and it has been studied on both continuous-time [4] and discrete-time dynamic agent systems [5–9]. The network topology is always invariant, and time variant topologies also have been paid much attention to in multi-agent problems. Yang et al. [10,11] have studied multi-agent systems under the switching topologies. The leader-following consensus problem of multi-agent systems is also an active topic and has been studied by many researchers. Optimal tracking problem has been studied in [12] based on the adaptive dynamic programming method. Pinning control was proposed in [13] to guarantee that the agents asymptotically follow the virtual leader in each group, while agents in different groups behave independently. Hong et al. [14] investigated a systematical framework of tracking control problem with an unmeasured active leader and proposed an "observer" by inserting an integrator into the loop for each agent to estimate the leader's velocity. Zhao et al. [15] extended the results of Hong et al. [14] and studied the distributed consensus tracking problem for multi-agent systems with Lipschitz-type dynamics.

On the other hand, output regulation is one of the most important and widely studied problems in control systems. Pioneering works are found in [16–18]. In recent years, output regulation of multi-agent systems had received considerable attention which is to control more than one plant so that their outputs track a reference signal(and/or rejects a disturbance) produced by an exosystem. Synchronized output regulation about identical subsystems was solved by Xiang et al. [19]. Xiang

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et al. [19] firstly regulated the tracking errors so that the errors converged to the synchronous manifold and then analyzed the stability of synchronous manifold. However, a more relaxable condition was given in [20] in which a distributed compensator was designed for the exosystem. Huang et al. [21] solved the output regulation of identical certain and uncertain linear multi-agent systems if a Riccati equation with parameters has a positive definite symmetric solution. Internal model principle was used in solving the robust output regulation for uncertain multi-agent systems such as [22–26]. Output regulation for heterogeneous uncertain system dynamics was considered in [22] in which the communication topology contains no loop. Wieland et al. [24] gave a necessary and sufficient condition for the output synchronization problem. It used the internal model method to solve the consensus with leaderless by setting up a virtual leader.

In this article, we give a general result about output regulation of uncertain heterogeneous multi-agent systems. The subsystems have non-identical nominal dynamics and also have the different uncertain parts. Since not all the agents can access the information of the exosystem, the distributed compensators are used for the unaccessible part. The benefits of this paper are introduced as threefold. First, our results are natural and relax significantly the condition in [22,27]. Second, it makes easier the method in solving the robust output regulation for heterogeneous multi-agent systems. At last, in the expression of the closed loop system, we use a new compact form in which the variables of compensators are composed with the variable of exosystem. Thus the traditional internal model criterion is invalid, and a novel global method is used in the proofs of Theorems 1 and 2.

The remainder of this article is organized as follows. Some preliminaries on graph theory and the system models are given in Sections 2 and 3. The main results based on dynamic state feedback and dynamic output feedback control are given in Section 4. In Section 5, numerical simulations are given to illustrate the effectiveness of theoretical results. Finally, some conclusions are presented in Section 6.

Notations: Throughout this article, let R, R^n and $R^{n\times m}$ denote the set of real numbers, n-dimension Euclidean space and the set of $n\times m$ real matrices. Denote $\lambda(A)$ be the eigenvalues of A. Notation $\otimes$ represents the Kronecker product. $\mathbf{0}$ denotes a matrix which has appropriate dimensions, and all the elements are 0. $\mathbf{1}_n$ represents the n-dimension vector with all entries being one.

2. Preliminaries

Some basic knowledge about graph theory is described as follows (For more details, one can refer to [28]):

The topology structure of a communication network can be expressed by a digraph. A weighted digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ is composed by a vertex set $\mathcal{V}=\{v_0, v_1, v_2,\ldots,v_N\}$, an edge set $\mathcal{E}=\{e_{ij}=(v_i,v_j)\}\subset\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]$ with nonnegative adjacency elements a_{ij}. Then, the Laplacian corresponding to the digraph $\mathcal{G}$ is defined as $L=[l_{ij}]$, where $l_{ij}=-a_{ij}, i\neq j$, and $l_{ii}=\sum_{j=1}^{N}a_{ij}$. $v_i, (i=1,\ldots,N)$ represents ith agent and v_0 represents the exosystem in this paper. An edge (v_i,v_j) in digraph $\mathcal{G}$ means that the agent v_i receives information from agent v_j. $a_{ij}>0$ if and only if $(v_i,v_j)\in\mathcal{E}$; other else, $a_{ij}=0$. The set of neighbors of agent v_i is denoted by $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$. A directed path from v_i to v_j in digraph $\mathcal{G}$ is a sequence of edges $(v_i,v_{i_1}),(v_{i_1},v_{i_2}),\ldots,(v_{i_m},v_j)$ with distinct nodes $i_k, k=1,\ldots,m$. A directed graph contains a directed spanning tree if there exists at least one agent which is called root node that has a directed path to every other agents.

Assuming that the graph $\overline{\mathcal{G}}=(\overline{\mathcal{V}},\overline{\mathcal{E}},\overline{\mathcal{A}})$ with vertex set $\overline{\mathcal{V}}=\{v_1,\ldots,v_N\}$ is the subgraph of digraph $\mathcal{G}$. The weighted adjacency matrix of $\mathcal{G}$ is expressed as follows:

$$\mathcal{A}=\begin{pmatrix} 0 & \mathbf{0} \\ \mathcal{A}_0\mathbf{1}_N & \overline{\mathcal{A}} \end{pmatrix}$$

where $\mathcal{A}_0=diag(a_{10},a_{20},\ldots,a_{N0})$, $\overline{\mathcal{A}}$ is the adjacency matrix of $\overline{\mathcal{G}}$. Then Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is:

$$\begin{pmatrix} 0 & \mathbf{0} \\ -\mathcal{A}_0\mathbf{1}_N & \mathcal{A}_0+\overline{\mathcal{L}} \end{pmatrix}$$

where $\bar{\mathcal{D}}=diag\{\sum_{j=1}^{N}a_{1j},\sum_{j=1}^{N}a_{2j},\ldots,\sum_{j=1}^{N}a_{Nj}\}$ and $\bar{\mathcal{L}}=\overline{\mathcal{D}}-\overline{\mathcal{A}}$ is the Laplacian matrix of subgraph $\bar{\mathcal{G}}$.

3. Problem formulation

In this paper, consensus output regulation problem of multi-agent systems have been considered and a group of agents has the following form:

$$\begin{cases} \dot{x}_i=\overline{A}_ix_i+\overline{B}_iu_i+\overline{E}_i\omega, \\ y_i=\overline{C}_ix_i, \qquad\qquad \mathrm{i}=1,\ldots,\mathrm{N}. \end{cases} \tag{1}$$

where the state $x_i\in R^n$ and the measured output $y_i\in R^p$, $u_i\in R^m$ is the consensus protocol to be designed later. $\overline{E}_i\omega$ is the disturbance of the ith agent to be rejected and ω is generated by the following system which is called exosystem:

$$\begin{cases} \dot{\omega}=\Gamma\omega, \\ y_r=Q\omega, \end{cases} \tag{2}$$