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A bioeconomic modeling of two-prey and one-predator fishery model with optimal harvesting policy through hybridization approach

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ABSTRACT

This paper introduces an imprecise prey–predator model by using interval and fuzzy number in modeling of prey and predator interaction. In a prey–predator system, values of the biological parameter are based on the collection of data by the experts from nature; therefore we consider them as interval numbers. The proposed prey–predator model is consisting of two species which are not only affected by the harvesting of the species but also by the present of predator species. All possible existence of the biological and bionomic equilibrium points of the model are discussed under impreciseness. We derive the conditions for local and global stability. The optimal harvesting policy is studied by considering instantaneous annual rate of discount under fuzziness. By logically hybridizing the interval and fuzzy number methodologies, we find the optimal equilibrium points and harvesting efforts in the form of interval. Finally, numerical examples are illustrated to support the proposed approach.

1. Introduction

Research in the area of theoretical ecology was started by Lotka [1] and Volterra [2]. Among the various bioeconomic modeling (Bhattacharya and Begum [3], Purohit [4], Samanta [5]) of exploitation of biological resources, fisheries and forestries has became most important. Clark [6,7] studied extensively bioeconomic exploitation of these resources. The biological resources are renewable resources which has great impact in our daily life. Now we are facing several problems due to the shortage of biological resources. The main reason behind that the extensive and unregulated harvesting of marine fish, which lead to the extinction of several species. Also the modern technology in fishing power, rate of increase world population and lack of knowledge of the benefits of the exploited species among the people are all causes of exploitation of different species of fishes. Therefore to protect the exploitation of different species of fishes optimal harvesting policy is very essential.

Presently, several researchers (Hannesson [8], Li and Wang [9], Li et al. [10], Murphy and Smith [11], Palma and Olivares [12], Guttormsen et al. [13], Luo et al. [14]) concentrate their research work in the field of optimal management of renewable resources. Kar and Chaudhuri [15] studied a prey–predator combined harvesting model of two competing species in presence of a predator. Kar et al. [16] presented a bioeconomic two prey one predator harvesting model by using two different

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functional responses of Holling types I and II. The optimal harvesting policy based prey–predator models ware presented by Chakraborty et al. [17] Gohary and Bukhari [18], Luo et al. [19], Zhang et al. [20] along with others.

Most of the scientists and researchers considered their mathematical models under precise environment i.e., they assumed that all the parameters and initial conditions or populations are precise in nature. However in real world problems are saturated with impreciseness. It is therefore quite natural to try to identify role of the impreciseness of the biological parameters in the behavior of population communities. In fact, many parameters may oscillate simultaneously with the periodically varying environments in real-world ecosystems. They are also varying due to both nature and human society, such as fire, earthquake, climate warming, financial crisis, etc. Therefore the interaction process between the species and its dynamics are strongly influenced by these environmental variations. The impreciseness can also take place in the experimental part, data collection, the measurement process, as well as determining the initial conditions. This phenomenon can be analyzed incorporating the model with imprecise biological parameters. To tackle such type of model with imprecise biological parameters there are different approaches such as stochastic approach, fuzzy approach and fuzzy-stochastic approach. There are very few literatures of bio-mathematical models based on impreciseness. Bassanezi et al. [21] put foundation stone to study the stability of a dynamical system using fuzzy differential equations. Barros et al. [22] presented environmental fuzziness of a life expectancy model by considering the parameters are fuzzy in nature. Peixoto et al. [23] considered predator–prey model under fuzziness. Mizukoshi et al. [24] studied the fuzzy initial value problem with parameters and/or initial conditions under fuzziness. Guo et al. [25] studied logistic model and Gompertz model under fuzziness. Recently Pal et al. [26] studied proportional harvesting model with fuzzy intrinsic growth rate and fuzzy harvesting quantity. On the other hand, to tackle environmental variations stochastic approach widely used by Alvarez [27], Liu and Wang [28], Ji et al. [29] Yagi and Ton [30], Liu and Bai [31] and the references there in. But in stochastic approach the imprecise parameters are replaced by random variables with known probability distributions. In fuzzy approach the imprecise parameters are replaced by fuzzy sets with known membership function or by fuzzy numbers. On the contrary, in fuzzy stochastic approach some parameters are taken as fuzzy in nature and rest of the parameters are taken as random variables. However it is very difficult to construct a suitable membership function or a suitable probability distribution for the imprecise biological parameters. To overcome these difficulties in fuzzy as well as stochastic approach Pal et al. [32] first time presents the concept of interval number to a simple Lotka Volterra type imprecise prey predator harvesting model. However to our knowledge, yet no attempt has been made to study the harvesting model of two competing species in presence of a predator under impreciseness of the biological parameters where the third species (predator) is not harvested. This paper consists of two preys which are continuously harvested by harvesting agencies and one predator which depending on the two prey species. Due to the various demand of the two prey species in the market we adopt two different harvesting techniques for the preys. We assume all the biological parameters involved in the model are imprecise in nature and represent them by the interval number. Using the parametric functional form (Mahapatra and Mandal [33]) of the interval number we find the different characteristics of the model such as biological equilibrium points, bionomic equilibrium points, stabilities of the equilibrium points etc. In addition, due to the responsibility to the society and necessity of preserving all the species we perform an optimal harvesting policy by taking instantaneous annual rate of discount as triangular fuzzy number. By logically hybridizing the interval number and fuzzy number methodologies, we find the optimal equilibrium points and harvesting efforts in the form of interval using Pontryagin's maximum principle. An excellent example of our model is minke whales as predator and two of its prey species named as juvenile herring and capelin. Lastly, numerical examples are given to support the proposed approach.

The rest of the paper is presented as follows: The prerequisite mathematics is discussed in Section 2. In Section 3 we present the model in crisp and imprecise environment. Section 4 presents the steady states of the imprecise model. Local and global stabilities are presented in Section 5. Existence of bionomic equilibrium is discussed in Section 6. The optimal harvesting policy is presented in Section 7. Numerical examples are illustrated in Section 8. Some conclusions are made in Section 9.

2. Prerequisite mathematics on interval and fuzzy number

In order to develop prey–predator mathematical model, we introduce certain preliminary definitions and results which will be used letter on.

2.1. Parametric interval arithmetic

An interval number A is symbolized by the closed interval $[a_L, a_R]$ and defined by $A = [a_L, a_R] = \{x : a_L \leqslant x \leqslant a_R,\ x \in \mathbb{R}\}$. Where $\mathbb{R}$ is the set of real numbers and a_L and a_R denote the left and right limits of the interval number respectively.

Definition 1. *Interval-valued function:* Let us consider an interval $[a_L, a_R]$, where $a_L > 0$. If the interval is of the form $[a_L, a_R]$, the interval-valued function is defined by $a(p) = a_L^{1-p} a_R^{p}, p \in [0, 1]$.

An interval can be represented by a function. Now we present some interval arithmetic operations based on interval-valued function of the interval as follows:

Let $A = [a_L, a_R]$ and $B = [b_L, b_R]$ be two interval numbers, then

Addition: $C = A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$, provided that $a_L, b_L > 0$. The interval-valued function for the interval $C = A + B$ is given by $c(p) = c_L^{1-p} c_R^{p}$, where $c_L = a_L + b_L$ and $c_R = a_R + b_R$.