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# Padé-type approximation method for two dimensional Fredholm integral equations of the second kind ☆

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ABSTRACT

Two dimensional Fredholm integral equation of the second kind (2DF-II) on a bounded domain $D$ is regarded as the problem with characteristic values. A two dimensional function-valued Padé-type approximation(2DFPTA) is defined. Its error formulas and convergence theorems are presented. To obtain higher order 2DFPTA, a determinantal expression and its recursive algorithm are given. In the end three numerical examples are tested, where one on the unit triangle of vertices $(0,0),(0,1),(1,0)$ and the other two on the square. The testing results show that 2DFPTA method is more accurate.
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## 1. Introduction

In [1] the authors transform a two dimensional transport equation into an two dimensional Fredholm integral equation of the second kind(2DF-II)

$$u(\mathbf{x}) = f(\mathbf{x}) + \lambda \int_D K(\mathbf{x},\mathbf{y})u(\mathbf{y})\omega(\mathbf{y})\mathrm{d}\mathbf{y}, \mathbf{x},\mathbf{y} \in D, \tag{1}$$

where $D$ is a bounded domain, $\mathbf{x} = (x_1,x_2), \mathbf{y} = (y_1,y_2),\ \mathrm{d}\mathbf{y} = \mathrm{d}y_1\mathrm{d}y_2, K$ and $f$ are given functions defined on $D, \omega$ is a weight function, $\lambda$ is a real parameter(called a characteristic value). 2DF-II is a useful tool to model many problems arising in fracture mechanics, aerodynamics, 2D electromagnetic scattering [2] and computer graphics manipulations [3]. For domain $D$, some works treat the square case [4–11] and some are based on piecewise approximating polynomials [8], Monte Carlo methods [7], or discrete Galerkin method [6], Nyström methods based on cubature rules obtained as the tensor product of two univariate Gaussian rules [9,11]. Padé-type method is a rational approximating technique, which is used to solve one dimensional Fredholm integral equations of second kind [12,13].

In this work our goal is to seek the solution of 2DF-II. We first regard $u(\mathbf{x})$ as a function with respect to the characteristic value $\lambda$ in (1), mamely $u(\mathbf{x}) = u(\mathbf{x},\lambda)$. Hence (1) is in the form

$$u(\mathbf{x},\lambda) = f(\mathbf{x}) + \lambda \int_D K(\mathbf{x},\mathbf{y})u(\mathbf{y},\lambda)\omega(\mathbf{y})\mathrm{d}\mathbf{y}, \mathbf{x},\mathbf{y} \in D, \tag{2}$$

In (2), the solution corresponding $\lambda$ is called a characteristic function.

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Set $u(\mathbf{x},\lambda)$ be analytic and meromorphic in a neighborhood $\widetilde{D}$ of the origin $\lambda = 0$. Let its Neumann series be given by

$$u(\mathbf{x},\lambda) = \sum_{i=0}^{\infty} c_i(\mathbf{x})\lambda^i, \tag{3}$$

where $c_i(\mathbf{x})$ continuous on $D$,

$$c_0(\mathbf{x}) = f(\mathbf{x})$$
$$c_i(\mathbf{x}) = \int_D K(\mathbf{x},\mathbf{y})c_{i-1}(\mathbf{y})\mathrm{d}\mathbf{y}, \text{for} i = 1,2,\cdots.$$

## 2. Padé-type approximation

Let $u(\mathbf{x},\lambda)$ be the Neumann series in $\lambda$ in the form (3). Let $c^{(l)} : \mathbf{P} \to \mathbb{C}$ be a linear functional on the polynomial space $\mathbf{P}$:

$$c^{(l)}(t^i) = c_{l+i}(x_1,x_2), \quad i = 0,1,\cdots, l \in \mathbb{Z}, \tag{4}$$

where $c^{(l)}(t^i) = 0$ for $l + i < 0$.

Let $|t\lambda| < 1$ and from the linear functional $c = c^{(0)}$ in (4). It follows that

$$c((1-t\lambda)^{-1}) = c(1 + t\lambda + (t\lambda)^2 + \cdots) = \sum_{i=0}^{\infty} c_i(x_1,x_2)\lambda^i = u(\mathbf{x},\lambda).$$

Let $v_n(\lambda) \in \mathbf{P}_n$ be a scalar polynomial of degree $n$

$$v_n(\lambda) = b_0 + b_1\lambda + \cdots + b_n\lambda^n, \tag{5}$$

where $b_n \neq 0$.

Define a polynomial $w_m(\mathbf{x},\lambda)$ in $\lambda$ by

$$w_m(\mathbf{x},\lambda) = c\left(\frac{t^{m-n+1}v_n(t) - \lambda^{m-n+1}v_n(\lambda)}{t-\lambda}\right). \tag{6}$$

It is clear that $w_m(\mathbf{x},\lambda)$ is of degree $m$ in $\lambda$.

Let us define

$$\tilde{v}_n(\lambda) = \lambda^n v_n(\lambda^{-1}) = \sum_{j=0}^{n} b_j\lambda^{n-j}, \tag{7}$$

$$\tilde{w}_m(\mathbf{x},\lambda) = \lambda^m w_m(\mathbf{x},\lambda^{-1}). \tag{8}$$

It is found from (7) that $b_n \neq 0$ implies $\tilde{v}_n(0) \neq 0$.

From (5), (6) and (8), $\tilde{w}(\mathbf{x},\lambda)$ is of the following form:

$$\tilde{w}_m(\mathbf{x},\lambda) = \sum_{j=0}^{n} b_j\lambda^{n-j}\sum_{i=0}^{m-n+j} c_i(\mathbf{x})\lambda^i = \sum_{i=0}^{m}\sum_{j=0}^{n} b_j\tilde{c}_i^j(\mathbf{x})\lambda^i, \tag{9}$$

where $\tilde{c}_i^j(\mathbf{x}) = c_{i-n+j}(\mathbf{x})$.

Using (7) and (9), we have

$$\begin{aligned}\tilde{v}_n(\lambda)u(\mathbf{x},\lambda) - \tilde{w}_m(\mathbf{x},\lambda) &= \left(\sum_{j=0}^{n} b_j\lambda^{n-j}\right)\left(\sum_{i=0}^{\infty} c_i(\mathbf{x})\lambda^i\right) - \sum_{j=0}^{n} b_j\lambda^{n-j}\sum_{i=0}^{m-n+j} c_i(\mathbf{x})\lambda^i = \sum_{i=0}^{\infty}\left(\sum_{j=0}^{n} b_j c_{m-n+1+i+j}(\mathbf{x})\right)\lambda^{m+1+i} \\ &= \sum_{i=m+1}^{\infty}\left(\sum_{j=0}^{n} b_j\tilde{c}_i^j(\mathbf{x})\right)\lambda^i = \mathcal{O}(\lambda^{m+1}).\end{aligned} \tag{10}$$

**Definition 1.** Let $\tilde{w}_m(\mathbf{x},\lambda)$ and $\tilde{v}_n(\lambda)$ be given by (9) and (10) respectively. Then the following rational function

$$r_{m,n}(\mathbf{x},\lambda) = \frac{\tilde{w}_m(\mathbf{x},\lambda)}{\tilde{v}_n(\lambda)} = \frac{\sum_{j=0}^{n} b_j\lambda^{n-j}\sum_{i=0}^{m-n+j} c_i(\mathbf{x})\lambda^i}{\sum_{j=0}^{n} b_j\lambda^{n-j}} = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} b_j\tilde{c}_i^j(\mathbf{x})\lambda^i}{\sum_{j=0}^{n} b_j\lambda^{n-j}}$$

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