Applied Mathematics and Computation 246 (2014) 88–102

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Zero dynamics of sampled-data models for nonlinear multivariable systems in fractional-order hold case

Cheng Zeng [a,b], Shan Liang [a,*]

[a] College of Automation, Chongqing University, Chongqing 400044, China
[b] College of Science, Guizhou Institute of Technology, Guiyang 550003, China

ARTICLE INFO

Keywords:
Nonlinear multivariable system
Sampled-data models
Zero dynamics
Fractional-order hold
Stability

ABSTRACT

The paper is concerned with the properties of approximate sampled-data models and their zero dynamics, as the sampling period tends to zero, composed of a fractional order hold (FROH), a continuous-time multivariable plant and a sampler in cascade. The emphasis of this paper is the stability of discrete zero dynamics with the generalized gain β of the FROH, where we also present a condition to assure the stability of the sampling zero dynamics, which they have no counterpart in the underlying continuous-time system, of the resulting model. Similar to the linear case, the parameter β is the only factor in affecting the stability of discrete zero dynamics, and the appropriate β is determined to obtain the FROH that provides zero dynamics as stable as possible, or with improved stability properties even when unstable, for a given continuous-time multivariable plant. The study is also shown that the stability of the sampling zero dynamics is improved compared with a zero-order hold (ZOH).

1. Introduction

Technology advances in digital electronics have led to a rapid development in computer technology. Currently, digital computers can be found in most equipment in a range of different applications. Control engineering is one of many areas where digital computer technology has made a great impact. Indeed, computer-controlled systems are now a prevalent configuration used in practice. A sampled-data system involves both continuous-time and discrete-time signals in the operation. By controlling a continuous-time plant using a digital controller that operates in a discrete-time environment, we form a sampled-data system. Consequently, a sampled-data control system is often referred to as a computer-controlled system [1,2].

For linear system, we can write down an exact sampled-data model while typically for nonlinear systems we can not. Moreover, the exact discrete-time model of a linear system is linear while the exact sampled-data model for a nonlinear system does not usually preserve important structures of the nonlinear systems [3]. However, most plants and processes are nonlinear in nature and there is a wide area of applications for sampled-data control systems, where nonlinear phenomena can not be avoided. Control of nonlinear sampled-data model is, therefore, an important area of control engineering with a range of potential applications. Furthermore, the theory for the sampled-data nonlinear models is less well developed than for linear case and the absence of good models for sampled-data nonlinear plants is still recognized as an important issue for control design [4,5]. For example, it is well known that better sampled-data nonlinear models can be generated by including

* Corresponding author.
E-mail addresses: zengcheng1290@163.com (C. Zeng), lightsun@cqu.edu.cn (S. Liang).

http://dx.doi.org/10.1016/j.amc.2014.08.001
0096-3003/

extra zero dynamics (i.e. "sampling zero dynamics") due to the sampling process in the nonlinear case while the famous Euler model is difficult to obtain good control performance because of the poor approximation.

Sampling zero dynamics, which correspond to the sampling zeros in the linear case [6], play an important role for analysis and design of nonlinear control systems. More specifically, the occurrence of nonlinear zero dynamics is relevant to the problem of control of nonlinear systems. For linear systems, the presence of sampling zeros has been deeply discussed in many papers following [6–19]. One would reasonably expect similar results about sampling zero dynamics to hold for nonlinear sampled-data systems. However, the situation for the nonlinear case is more complex than that of the linear systems. In the nonlinear case, there is a difficulty in essence that the exact sampled-data nonlinear model does usually not preserve important structures of the nonlinear systems while the exact discrete-time model of a linear system is linear [3].

In the single-input single-output (SISO) case, the nonlinear results by Yuz and Goodwin, and Ishitobi et al. have been shown in the case of a zero-order hold (ZOH) [20–23]. They have proposed a more accurate approximate model than the simple Euler model. The resulting model includes extra zero dynamics which are called sampling zero dynamics. It has been shown explicitly that they have no counterpart in the underlying continuous-time system and are the same as those for linear case [6], although an implicit characterization has been given in [24]. In addition, Ishitobi and Nishi [25,26] have also showed that the stability of zero dynamics will be improved by using fractional order hold (FROH) stead of ZOH. When the relative degree of a continuous-time system is two, use of a FROH overcomes the problem of the instability of the sampling zero dynamics in the case of a ZOH [25,26].

The SISO results in the case of a ZOH have been extended to multi-input multi-output (MIMO) nonlinear systems [27]. A sampled-data model called Yuz and Goodwin type model is derived when the multivariable nonlinear system is decouplable by static state feedback, and the sampling zero dynamics of the resulting model are analyzed. It is, however, pointed out that the Yuz and Goodwin type model can not be used for discrete-time controller design because of the existence of the unstable sampling zero dynamics in some case when at least one of the relative degrees of a continuous-time multivariable nonlinear plants are equal to two. The reason is that the closed-loop system becomes unstable when a discrete-time controller design method based on the assumption of the stability of the zero dynamics can be applied. Therefore, the stability of the closed-loop system depends on directly stability of the sampling zero dynamics of the sampled-data model.

A comparative study has demonstrated that a FROH provides an superior advantage over a ZOH as far as the stability of zero dynamics of the resulting discrete-time systems is concerned. So far, the properties of zero dynamics of the discrete-time nonlinear systems with FROH are discussed mainly on SISO case. Hence, it is natural that one has the following questions: how can the results of the ZOH case be extended to a FROH case for decouplable nonlinear multivariable systems? Does FROH always provide an advantage over ZOH similar to those of nonlinear SISO systems or linear cases? How does the superiority of FROH to ZOH depend on the parameter β in the case of nonlinear multivariable system?

The purpose of this article is to solve these problems and to analyze the resulting sampled-data models and their zero dynamics in the case of a FROH in detail. In this paper, we first derive an approximate sampled-data model for nonlinear multivariable system with FROH, which the local truncation errors of proposed model between the outputs of this model and the true continuous-time system outputs are of order T^{r_i+1}, where T is the sampling period and r_i is one of the relative degrees of nonlinear system. Next, we analyze zero dynamics of the sampled-data model with FROH to show a condition which ensures the stability of the zero dynamics of the model proposed. More importantly, this has an advantage of giving insight into the sampling zero dynamics and also stability criteria for sampling zero dynamics, which are a new addition owing to sampling process and turn out to be identical to those found in the linear case, can be obtained in terms of relative degrees less than or equal to two in the case of a FROH. Further, we design a discrete-time model following controller for a continuous-time multivariable plant on the basis of the sampled-data model, and apply it to the original continuous-time system through a FROH. Our results with FROH reveal that the convergence of the output to the origin is achieved, while the output does not converge to the origin through a ZOH. Moreover, the ideas presented here generalize well-known results from the nonlinear SISO case to multivariable models, and also from the linear systems to nonlinear plants. It has definitely shown that the FROH is superior to the ZOH in both the stability of zero dynamics for nonlinear MIMO systems and the discrete-time model following controller.

2. System description

Consider an m-input m-output nth-order square nonlinear continuous-time systems decouplable by static state feedback. It is assumed that the system has the uniform relative degrees $r_1, \ldots, r_m$. Then, the system can be expressed in its so-called normal form [28,29]

$$\begin{cases} \dot{\zeta}^i = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{r_i-1} \\ 0 & \mathbf{0}^T \end{bmatrix} \zeta^i + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \left(b_i(\boldsymbol{\zeta}, \boldsymbol{\eta}) + \sum_{j=1}^{m} a_{ij}(\boldsymbol{\zeta}, \boldsymbol{\eta}) u_j \right), \quad i = 1, \ldots, m, \\ \dot{\boldsymbol{\eta}} = c(\boldsymbol{\zeta}, \boldsymbol{\eta}), \\ y_i = \zeta_1^i, \ i = 1, \ldots, m, \end{cases} \tag{1}$$