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Global stability for a sheep brucellosis model with immigration

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ABSTRACT

Sheep brucellosis is one of the major infectious and contagious bacterial diseases in the sheep farms of China. In this paper, we present a sheep brucellosis model with immigration and proportional birth, and consider both direct and indirect transmission with infected animals and the bacteria of the environment. The basic reproduction number R_0 of this model is identified and global dynamics are completely determined by R_0. If $R_0 < 1$, the disease-free equilibrium is global asymptotically stable; whereas if $R_0 > 1$, there is a unique endemic equilibrium which is global asymptotically stable. By numerical simulations for the cases with $R_0 < 1$ and $R_0 > 1$ to demonstrate the global stability of the disease-free equilibrium and the unique endemic equilibrium, respectively. In addition, sensitivity analysis of the basic reproduction number in term of some parameters is given, this paper confirmed that elimination, vaccination and disinfection are the useful control strategies, and proposed to reduce immigration and self-sufficiency of the flock for controlling sheep brucellosis.

1. Introduction

Brucellosis, also known as undulant fever, Mediterranean fever or Malta fever is a zoonosis and the infection is almost invariably transmitted by direct or indirect contact with infected animals or their products [6]. Brucellosis is one of the animal diseases, especially domesticated livestock, which is caused by bacteria of the Brucella group. The bacteria of the genus Brucella has six species: *Brucella abortus*, *Brucella suis*, *Brucella melitensis*, *Brucella neotomae*, *Brucella ovis* and *Brucella canis* [7]. Brucellosis is usually caused by *B. abortus* in cattle, *B. melitensis* in sheep and goats, *B. ovis* in small ruminants, *B. suis* in pigs and *B. canis* in dogs. The survival time of brucella ranges from one to four months in the contaminated soil and water, and two months in milk and meat. However, it is easily killed by direct sunlight, high temperature and effective disinfectant [23]. Brucellosis is a sub-acute or chronic disease which may affect many species of animals, especially cattle, sheep, goats, pigs [21]. Infections in sheep is highly contagious because of the pathogenicity of B. melitensis and the infected sheep exhibit symptoms which may include abortion during the last third of pregnancy, retained afterbirth, and weak calves at birth. The brucellosis organism is shed into the environment by the millions in the afterbirth and fluids associated with calving and aborting, so brucella also widely exists in the environment near or in the farms and it leads to indirect transmission about

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brucella-to-sheep for brucellosis. In this paper, we will consider sheep brucellosis with indirect transmission by the bacterium in the environment around.

In China, Brucellosis infection in sheep will cause losses of abortion and thus it brought about serious economic consequences for the sheep industry. Control measures for brucellosis with livestock include quarantine, livestock vaccination and the elimination of infected animals [22]. With the development of animal husbandry, there was a significant increase for the number of animals from 1987 to 2007 in China, the number of sheep increased 2 times respectively. However, vaccination rate dropped from 99.71% to 50% or less, which lead to sheep positive rate to be 1.49% in 2009 [13]. Therefore it is significative to investigate sheep brucellosis systematically in the sheep farms of China.

In the past decade, various mathematical models have been proposed and analyzed to understand the dynamics of spread of infectious diseases [3,19,20,27,24,29–35]. In order to explore the transmission rules and find effective control strategies for sheep brucellosis, several mathematical dynamic models have also been developed to study the transmission of brucellosis [13,37,2,17]. For example, Hou et al. [13] proposed a dynamic model which included transmission within sheep and the transmission from sheep to humans for brucellosis in Inner Mongolia of China. Zinsstag et al. [37] represented a cattle-sheep-human brucellosis transmission dynamic model on the characteristics of Mongolia. Alnseba et al. [2] studied an susceptible, infected and the contaminated environment dynamical model for brucellosis epidemic in ovine with direct and indirect transmission. Li et al. [17] investigated a multi-group dynamical model with bidirectional mixed cross infection between cattle and sheep in public farms of China. Based on previous works and the facts of brucellosis infection in the sheep farms, in this paper we propose a deterministic dynamical model to investigate sheep brucellosis systematically in the sheep farms of China.

The article is organized as follows. In Sections 2, we present and interpret the dynamical model that describes the brucellosis transmission between sheep in the sheep farms of China, give the basic reproduction number of the model and some mathematical analysis. And in Section 3, some numerical simulations are showed to illustrate the effectiveness of the proposed result. In Section 4, we give a brief discussion about the main results.

2. Mathematical modeling and analysis

2.1. Model formulation

Based on the facts of brucellosis infection in the sheep farms, we classify the sheep population into four compartments: the susceptible compartment $S(t)$, the exposed compartment $E(t)$ (In this compartment for sheep, which is asymptomatic infectious), the infectious compartment $I(t)$ and the vaccinated compartment $V(t)$ at time t. Let $W(t)$ denote the quantity of sheep brucella in the environment at time t, it is only a assumption because it is very difficult to determine the quantity in environment and the quantity that is enough to infect an individual [17]. Thus, according to the paper [13], we also define the average number of brucella that is enough for a host to be infected as an infectious unit, so the unit of $W(t)$ is the infectious unit. Brucellosis can transmit to other sheep through directed contacting with exposed and infected sheep. On the other hand, exposed and infected sheep can shed brucella into the environment due to abortion or sheep secretions, so brucellosis can also transmit to other sheep through indirected transmission by brucella in the environment.

There are some assumptions and interpretations for our model. (i) From previous description of this paper, we know that the infected sheep exhibit symptoms which may include abortion, retained afterbirth, and weak calves at birth. Hence, the birth number of the compartment $E(t)$ and $I(t)$ can be ignored and we suppose the birth number of the sheep group per unit time is $b(S+V)$, b is the birth rate of the compartment $S(t)$ and $V(t)$. (ii) We assume that sheep in incubation period and infection period have the same transmission rate and discharge the same quantity of brucella into the environment per unit time. (iii) The vaccinated sheep cannot be infected unless they lose the immunity from the vaccination. There are also some other assumptions on the dynamical transmission of brucellosis in sheep are demonstrated in the flowchart.

From the Fig. 1 we can know that the new infection term is given by $\beta S(E+I) + \beta_1 SW$, the form of incidence is bilinear. $\beta S(E+I)$ is the direct infection from the exposed and infectious sheep to the susceptible sheep, $\beta_1 SW$ is the indirect infection from the brucella in environment to the susceptible sheep. Within the sheep group, it is assumed that the output occurs in $S(t), E(t), I(t)$ and $V(t)$ compartments with a rate of constant d. Individuals in $I(t)$ compartment suffer an additional death due

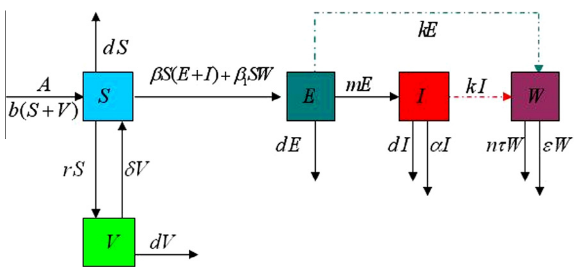

Fig. 1. Transmission diagram on the dynamical transmission of brucellosis among sheep.