



Exact and approximate solutions for the anti-symmetric quadratic truly nonlinear oscillator



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ABSTRACT

The exact solution of the anti-symmetric quadratic truly nonlinear oscillator is derived from the first integral of the nonlinear differential equation which governs the behavior of this oscillator. This exact solution is expressed as a piecewise function including Jacobi elliptic cosine functions. The Fourier series expansion of the exact solution is also analyzed and its coefficients are computed numerically. We also show that these Fourier coefficients decrease rapidly and, consequently, using just a few of them provides an accurate analytical representation of the exact periodic solution. Some approximate solutions containing only two harmonics as well as a rational harmonic representation are obtained and compared with the exact solution.

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1. Introduction

Nonlinear oscillator models have been widely used in many areas, not only in mechanics but also in other branches of physics, mathematics or engineering. There is a large variety of approximate methods commonly used for solving nonlinear oscillatory systems [1,2]. In conservative nonlinear oscillators the restoring force is not dependent on time, the total energy is constant [2,3] and any oscillation is stationary. An important feature of the solutions for conservative oscillators is that they are periodic and range over a continuous interval of initial values [4]. Conservative truly nonlinear oscillatory systems are modeled by differential equations for which the restoring force has no linear approximation at $x = 0$ [4]. In this paper we consider a conservative truly nonlinear oscillator for which the nonlinear function is an anti-symmetric quadratic function, $f(x) = \text{sgn}(x)x^2$. In recent years some examples of this class of truly nonlinear oscillators have been analyzed [4,5] and several techniques have been used to obtain analytical approximate solutions, such as harmonic balance, rational harmonic balance, parameter expansion, iteration or averaging methods. Recently, Cveticanin and Pogány [6] obtained an exact solution in the form of Ateb function for this class of conservative truly nonlinear oscillators. However, in the present paper we derive the exact solution of this type of oscillator and we obtain the Fourier series expansion of this exact solution. Some approximate solutions containing only two harmonics as well as a rational harmonic representation are obtained and compared with the exact solution. It may be concluded that the former are more accurate than the latter since after the second Fourier coefficient the signs of the coefficients alternate between negative and positive, whereas all the Fourier coefficients of the rational harmonic solution are positive.

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2. Formulation and solution method

The anti-symmetric quadratic truly nonlinear oscillator is a conservative single-degree-of-freedom oscillatory system governed by the following second-order differential equation

$$\frac{d^2x}{dt^2} + \operatorname{sgn}(x)x^2 = 0 \quad (1)$$

with initial conditions

$$x(0) = A \quad \frac{dx}{dt}(0) = 0 \quad (2)$$

where x and t are the non-dimensional displacement and time. This system corresponds to a truly nonlinear oscillator [4] and is an example of a non-smooth oscillator for which $f(x) = \operatorname{sgn}(x)x^2$ is a non-smooth function of the non-dimensional displacement x . The nonlinear function $f(x)$ is also odd, i.e. $f(-x) = -f(x)$ and satisfies $x f(x) > 0$ for $x \in [-A, A]$, $x \neq 0$, where $A > 0$ is the oscillation amplitude. All the solutions to the anti-symmetric quadratic oscillator are periodic [4] and this system oscillates around the equilibrium position $x = 0$ and the period, T , and periodic solution, x , are dependent on A .

The system equations are

$$\frac{dx}{dt} = y \quad \frac{dy}{dt} = -\operatorname{sgn}(x)x^2 \quad (3)$$

and the differential equation for the phase-plane trajectories is

$$\frac{dy}{dx} = -\frac{\operatorname{sgn}(x)x^2}{y} \quad (4)$$

Eq. (1) is a conservative system and has the following first integral

$$\frac{1}{2}y^2 + \frac{1}{3}\operatorname{sgn}(x)x^3 = \frac{1}{3}A^3 \geq 0 \quad (5)$$

which defines a family of bounded and simple closed curves in the phase-plane which are “oval” shaped as we can see in Fig. 1 [4]. The x -axis is the “zero” null-cline and the y -axis is the “infinite” null-cline. These null-clines divide the phase-plane into four open regions, each coinciding with a quadrant of the plane and in each quadrant, the sign of the derivative, dy/dx , has a definite value.

Let $u = x/A$, then we can rewrite Eqs. (1) and (2) as

$$\frac{d^2u}{dt^2} + A\operatorname{sgn}(u)u^2 = 0 \quad (6)$$

$$u(0) = 1 \quad \frac{du}{dt}(0) = 0 \quad (7)$$

Integrating Eq. (6) and using the initial conditions in Eq. (7), we arrive at

$$\frac{1}{2}\left(\frac{du}{dt}\right)^2 + \frac{1}{3}A\operatorname{sgn}(u)u^3 = \frac{1}{3}A \quad (8)$$

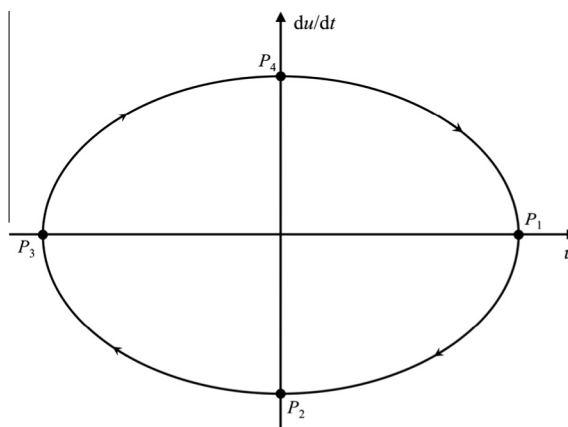


Fig. 1. Trajectories in the phase plane for the anti-symmetric quadratic nonlinear oscillator.

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