Applied Mathematics and Computation 246 (2014) 389–398

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Evolution of cooperation in the traveler's dilemma game on two coupled lattices

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ARTICLE INFO

Keywords:
Traveler's dilemma game
Evolution of cooperation
Network reciprocity
Interdependency
Coupled lattices

ABSTRACT

Although the evolutionary game theory provides a powerful framework to investigate various social dilemmas, the traveler's dilemma game (TDG) is hardly concerned under this platform within the applied mathematical communities. We put forward an improved traveler's dilemma game model on two coupled lattices to investigate the effect of coupling effect on the evolution of cooperation based on the traveler's dilemma game, where the coupling effect between two lattices is added into the strategy imitation process. Large quantities of simulations indicate that the cooperation behavior can be greatly varied when compared to those obtained on the traditionally single lattices. When the model parameter R surpasses a specific threshold, the cooperation will be greatly enhanced, but the cooperation may be inhibited if R is smaller than this threshold. Meanwhile, we also explore the relationship between the critical value R_c leading to the extinction of cooperation and the strategy range parameter *radius* in our model. Our results are surprisingly conducive to understanding the cooperation behavior of traveler's dilemma game within many real-world systems, especially for coupled and interdependent networked systems.

1. Introduction

Although the Darwinian theory of natural selection drives the evolution of species within many natural and social systems, the whole population can also benefit from the collective cooperation and sustain its persistent survival and development [1]. Thus, a dilemma arises and understanding the widespread phenomenon of cooperation becomes an interdisciplinary challenge from natural, biological, social and mathematical sciences [2–5]. Among them, evolutionary game theory [6] is often utilized as a powerful tool to illustrate the emergency of cooperation between unrelated and selfish individuals, in which the cooperation phenomenon, to a greater extent, can be accounted for through various game models including the prisoner's dilemma game (PDG) [7–13], snowdrift game (SDG) [14–18] and public goods game (PGG) [19–25] and so on. Beginning with these models, many quantitative and qualitative results assist in explaining the evolution of cooperation inside many real-worlds systems [3–5].

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http://dx.doi.org/10.1016/j.amc.2014.08.006
0096-3003/

Nevertheless, a distinct different game model-traveler's dilemma game (TDG)-receives a great deal of attention within the economics community. TDG is also a typical pair-wise game model which was first proposed by Basu in 1994 [26], and it can be briefly explained as follows: Two travelers have purchased the same souvenirs when they visit a remote place. After returning home by taking an identical flight, the airlines told them that their souvenirs had been lost and would provide some compensation for them. The representative of airlines, who does not know the exact value of souvenirs but realizes that the price lies between R and M ($R < M$), then asks each traveler to separately and privately declare the price of the souvenir. If both travelers claim the same prices, they will get that amount of money as a compensation from the airlines; But if they declare different prices, the representative believes that the lower price approaches the souvenir's actual value much more and provides the compensation to these two travelers with that smaller amount. Additionally, the airlines will also reward the traveler declaring the lower price with a bonus R and punish the traveler claiming the higher one with the same amount R. As a example, we consider a simple case in which R is assumed to be 4, and one traveler gives the price of his souvenir to be $i = 40$ while the other claims the value of his souvenir to be $j = 35$; Finally, the first traveler will receive $j - R = 31$ and the second one $j + R = 39$ for the compensation from the airlines, respectively. Theoretically, the Nash equilibrium of TDG model is that both travelers declare the minimal value R regardless of M, but the maximal total payoff may be up to $2M$ if both travelers claim the highest value M [26]. Accordingly, a typical social dilemma emerges for these two travelers in this context.

The traveler's dilemma game can also be used to model the human behaviors, and extensive experiments in the laboratory demonstrate that the players may behave to be totally different from the theoretical prediction. For example, Capra et al. [27] found a significantly inverse relationship between the average claim and R in TDG, which will hold both in early round before any in-game learning occurred and in later rounds after subjects gained experience; Some stochastically learning frameworks are also integrated into the TDG to enrich the understanding of human decision behaviors [28,29]. Meanwhile, in Ref. [30], players of TDG are placed into the spatial lattice in which each player can only interact with its nearest neighbors and imitate the strategy of neighboring agents according to unconditional imitation (UI) and Fermi rules, and the results indicate that the collective cooperation level is proportional to the average value of strategies. To be a further step, a self-questioning rule regarding much less information or lower game cost is proposed to update the individual strategy [31], and the cooperation can be enhanced onto a higher level when compared to Ref. [30].

However, the above-mentioned works often assume that the topology performing the TDG is well-mixing or an isolated spatial lattice, and hence it is far away from the real-world situations. It is well-known that real systems can often exhibit the small-world or scale-free properties [32], and graph representation and statistical analysis of real systems attracts a great deal of concern within the scientific communities [33–41]. In particular, even these systems are intercoupled and interdependent [42]. For instance, power grids and communication or information systems often depend on each other to support their normal behavior and function, and even lead to the so-called cascading failure created initially by a tiny malfunction or fault within either of both systems. Therefore, exploring the evolution of cooperation under interdependent and interacting circumstances becomes a challenging project at present, and several very recent works begin to investigate the cooperative behaviors under the PDG or PGG models, and it is found that the interdependency or multiplexing of interacting individuals is greatly positive for the emergency of cooperation [43–49]. Yet, the behavior of cooperation on the interdependent spatial lattices is less probed for the TDG model, and we will propose an interdependent or coupled TDG model under the stochastic learning framework in this paper. Here, we integrate the proportion of players on the other lattice falling within specific strategy ranges into the strategy update probability of focal player, it is clearly indicated that the cooperation behavior can be largely changed when compared to the TDG on a single lattice.

The remainder of this paper is structured as follows. At first, we will illustrate the proposed TDG model in detail in Section 2. Then, extensive numerical simulations are performed to characterize the influence of interdependency on the cooperation in Section 3. At last, Section 4 ends this paper with some concluding remarks.

2. Models

In this paper, we clarify the model of TDG on the coupled network which contains two identical $L \times L$ square lattices with periodic boundary condition and the interdependency (coupling or correlation) between these two lattices is assumed to be point to point. Here, we perform the TDG on the coupled network composed of two lattices (named after **UP** and **DOWN**) in which the actual link between corresponding nodes (*i.e.*, players) does not exist and the interdependency is implemented by the coupling of strategy update probability.

Firstly, we randomly select a network, let's say network **UP**, to start the game where each player is assigned to a random integer between R and M as his/her strategy, and hence each one has $M - R + 1$ possible strategies. The same procedure will be repeated for all agents on the other network (*e.g.*, network **DOWN**). We describe these strategies as $R, R+1, \ldots, M$, in which $1 < R < M$ must be satisfied. For a two-person game with multiple strategies in the TDG, the payoff matrix $A = (a_{ij})$, where $R \leqslant i, j \leqslant M$, and each entry a_{ij} is the payoff of one traveler declaring a value i when the other traveler declares a value j and a_{ij} is calculated as follows,