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Formation tracking of the second-order multi-agent systems using position-only information via impulsive control with input delays

Yan-Wu Wang [a,*], Meng Liu [a], Zhi-Wei Liu [b], Jing-Wen Yi [a]

[a] School of Automation, Key Laboratory of Image Processing and Intelligent Control of Education Ministry of China, Huazhong University of Science and Technology, Wuhan 430074, PR China

[b] Department of Automation, Wuhan University, Wuhan 430072, PR China

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ABSTRACT

In this paper, the formation tracking problem for the second-order multi-agent system with and without input delay are investigated, respectively. The objective is to design the formation tracking algorithm such that a certain follower follows the trajectory of the leader while also maintains a certain desired geometric formation with other agents simultaneously. The impulsive algorithms are designed by using only the relative position information of the neighbors for both the cases with and without the input delay. By using properties of the Laplacian matrix and combining the stability theory of impulsive systems, necessary and sufficient conditions are derived to achieve the formation tracking of the second-order multi-agent system with and without the input delay, respectively. The numerical examples are given to illustrate the effectiveness of our theoretical results.

1. Introduction

In recent years, the multi-agent system (MAS) has drawn significant attention owing to its various applications in many areas, such as sensor networks [1], multiple mobile robots [2], unmanned air vehicles [3]. The research topics of MAS include consensus [4–6], rendezvous [7,8], flocking [9,10], formation [11–14] and so on, among which formation control has been a hot topic [15,16]. It has appealed to more and more researchers due to its broad range of applications in autonomous unmanned vehicles, navigation of spacecrafts and the cooperative control of mobile robot systems. Formation control problem in the presence of a leader which is called formation tracking problem [17] can be seen as an extension of the general formation control problem.

Over the past decades, the formation tracking problems have been studied by many researchers. In the existing literatures, many different control methods have been utilized including the sliding mode control [18,19], adaptive control [20–22], model predictive control [23,24], feedback control [25] and so on. The decentralized sliding mode estimators were introduced to achieve decentralized formation tracking of multiple autonomous vehicles and multiple robot system in [18,19], respectively. The adaptive control laws were proposed to solve the formation tracking problem of electrically driven multiple mobile robots and multiple fully actuated surface vessels in [20,21], respectively. A distributed adaptive control approach based on backstepping technique was proposed to achieve the consensus tracking and the formation control of nonholonomic mobile robots in [22]. The model predictive control was adopted to study the formation control problem

* Corresponding author.
E-mail address: wangyw@mail.hust.edu.cn (Y.-W. Wang).

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for mobile robots in active target tracking problem in [23]. The model predictive control was proposed to solve the two sub-problems: trajectory control and formation control problems in [24]. The output-feedback cooperative control was applied to the formation tracking problem of unicycle-type mobile robots in [25].

The control methods mentioned above have the common feature of requiring continuous control. However, utilizing continuous control may result in much energy consumption, large amounts of communication bandwidth and short communication lifetime. Unlike the continuous control method, the impulsive control is only exerted at impulsive instants and has advantages over the above continuous control in some aspects, such as smaller control efforts, less energy consumption, less occupation of communication bandwidth and simpler implementation.

To the best of the authors' knowledge, there are no results concerning with the formation tracking of the second-order MASs by utilizing the impulsive control with or without input delays. Therefore, the formation tracking of second-order MAS with and without input delay via impulsive control are studied in this paper. Both the directed graph and undirected graph are considered. Considering that the velocity states of agents are usually difficult to be measured in practice and the position states of agents can be obtained more easily, instead, a novel impulsive algorithm with position-only information is proposed to achieve the formation tracking for the second-order MAS. The necessary and sufficient conditions are derived for the formation tracking of the MAS with and without input delay, respectively. These results give the guidelines to design the impulsive formation tracking algorithm for the second-order MAS. The numerical examples are given to illustrate our theoretical results.

The rest of the paper is organized as follows. Some preliminaries of algebraic graph theory and matrix theory, as well as the model description and the definition of formation tracking are given in Section 2. The convergence analysis of the proposed impulsive algorithms with and without input delay under both the directed and undirected graph are formulated in Section 3. The illustrative numerical examples are given in Section 4. Some conclusions are finally drawn in Section 5.

Notations: The notations used in this paper are fairly standard. $\mathbb{R}, \mathbb{C}$ and $\mathbb{N}_+$ denote the sets of real numbers, complex numbers and positive integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the space of $m\times n$-dimensional matrices with real entries, respectively. For $\gamma \in \mathbb{C}, \mathrm{Re}(\gamma)$ and $\mathrm{Im}(\gamma)$ are the real part and the imaginary of γ, respectively. The superscript 'T' stands for the transpose of a matrix. The notation $\|\cdot\|$ refers to the vector norm. The notation $\otimes$ denotes the Kronecker product of matrices. 1_N is a column vector with all its elements equal to one, 0_N is a column vector with all its elements equal to zero, I_N is a N-dimensional identity matrix, $diag\{\cdots\}$ refers to a diagonal matrix, or a block diagonal matrix. $\rho(P)$ denotes the spectral radius of square matrix P.

2. Preliminaries and problem formulation

In this section, some preliminaries of algebraic graph theory and matrix theory, as well as the model descriptions and the definition of formation tracking of the system are given.

To begin with, some basic concepts and lemmas on the algebraic graph theory and the matrix theory are briefly introduced.

A directed graph $G = (V, \varepsilon, A)$ is described with a set of nodes $V = \{1, 2, \ldots, N\}$, a set of edges $\varepsilon \in V \times V$ and the weighted adjacency matrix $A = (a_{ij})_{N\times N}$. Node i represents the $i-th$ agent, an ordered pair $\{j, i\}$ denotes an edge in G. $\{j, i\} \in \varepsilon$ if and only if the $i-th$ agent can obtain information from the $j-th$ agent directly. In this case, the $j-th$ agent is the neighbor of the $i-th$ agent. The set of neighbors of the $i-th$ agent is denoted by $N_i = \{j \in V | (j, i) \in \varepsilon\}$. The adjacency elements are nonnegative. For $j \in N_i$, $a_{ij} > 0$; and $a_{ij} = 0$, otherwise, assume that $a_{ii} = 0, i \in V$. A directed path in digraph G is an ordered sequence $\upsilon_1, \upsilon_2, \ldots, \upsilon_k$ of agents such that any ordered pair of vertices appearing consecutively in the sequence is an edge of the digraph, i.e., $(\upsilon_i, \upsilon_{i+1}) \in \varepsilon$, for any $i = 1, 2, \ldots, k-1$. A directed tree is a digraph, where there exists an agent, called the root, such that any other agent of the digraph can be reached by one and only one path starting at the root.

A graph $G = (V, \varepsilon, A)$ is undirected if $a_{ij} = a_{ji}, \forall i, j \in V$.

The Laplacian matrix $L = (l_{ij})_{N\times N}$ of graph G is defined as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum\limits_{k=1,k\neq i}^{N} a_{ik}, & i = j. \end{cases} \tag{1}$$

Lemma 1 [26]. *Let L be the Laplacian matrix of digraph G. Zero is a simple eigenvalue of L, and all the other eigenvalues have positive real parts if and only if G contains a spanning tree.*

Lemma 2 [27]. *The Laplacian matrix L of an undirected graph G is semi-positive definite. It has a simple zero eigenvalue and all the other eigenvalues are positive if and only if the graph G is connected, that is, all the eigenvalues of L satisfy* $0 = \lambda_1(L) < \lambda_2(L) \leqslant \cdots \leqslant \lambda_N(L)$.

Lemma 3 (Bilinear Transformation Theorem [28]). *Polynomial* $f(\lambda)$ *(of degree d) is Schur stable if and only if polynomial* $g(s)$ *is Hurwitz stable, where* $g(s) = (s-1)^d f(\frac{s+1}{s-1})$.