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Stability analysis of linear systems with two delays of overlapping ranges

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ABSTRACT

In this paper, a delay-dependent stability criterion is derived for linear time-delay systems with two constant delays having overlapping ranges. This overlapping feature of the delays is exploited in delay-dependent analysis of the system using Lyapunov–Krasovskii approach. The developed treatment of the delays reduces conservatism in analysis compared to the approaches treating the delays individually. Numerical examples are used to demonstrate the less conservativeness of the derived criterion.

1. Introduction

Time-delay is inherent to numerous engineering systems, such as process plants, chemical engineering systems and so on [1–3]. Delay in a system causes instability or degrades the system performance [1,4]. Therefore, stability analysis of such systems are important. Depending on whether the stability criteria depend on the delay value or not, they are classified as delay-independent and delay-dependent one. It is well known that the delay-dependent analysis is less conservative especially for smaller delay values than the delay-independent one [5–7]. Several delay-dependent stability criteria have been reported in literature for stability analysis of time-delay systems, see [7–18] and references therein.

For a linear time-delay system, a model transformation approach has been used in [19,20] and stability criteria are derived by applying some bounding technique for cross terms. As the transformed model adds some additional dynamics [1] that is not present in the original system, the resulting criteria may become conservative. To avoid this, a descriptor system model transformation has been developed in [21] to transform the original system to a descriptor form, then appropriate bounding technique for some cross terms are used to obtain less conservative results. In [22], Jensen's inequality is used to derive less conservative delay-dependent criterion by avoiding model transformation and bounding techniques. To obtain less conservative criteria, initiatives have been taken by using free-weighted matrix variables in deriving the criteria [23,14]. Further less conservative results have been obtained by splitting the delay interval in multiple regions, so that delay-dependent conditions can be imposed to a unique functional in the different regions. This interval approach leads to a effective reduction in conservatism [24,25,14].

Often multiple delays are encountered in practical systems [26]. For example, a feedback control loop may introduce additional delays while another delay embedded into the plant model itself [27,28]. Multiple delay example is also found

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in networked control systems with both sensor to controller and controller to actuator delay [29]. It may be noted that such multiple constant delays may have overlapping ranges due to their limited nature.

Existing attempts for stability analysis of systems with multiple time-delays are mere extensions of the approaches for single delay case. In such cases, the delays are treated individually in analysis [27,28,30]. This paper emphasizes on the development of a less conservative approach for systems with two delays by exploiting the overlapping delay range information. Though the approach is applicable for systems with multiple delays, the discussion will be limited for two delay case only. It is also considered that since the delays are limited they have overlapping ranges. As the overlapping information is concerned for two delay system, four different overlapping cases do arise for defining the Lyapunov–Krasovskii functional. Considering such multiple functional for different overlapping situations, a single stability criterion is derived that satisfies stability requirement of all the situations. This reduces the number of matrix variables involved in the criterion resulting in complexity reduction. On consideration of numerical examples, it is observed that the approach exploiting the overlapping feature yields less conservative result compared to the individual treatment of the delays.

Notations: Throughout this paper, the super script 'T' stands for matrix transposition, $\Re^n$ denotes the n-dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n \times m$ real matrices, and $P > 0$, for $P \in \Re^{n\times n}$, means that P is a symmetric and positive definite matrix. The symmetric terms in a matrix is denoted by $*$. I represents to Identity matrix with suitable dimension.

2. The system under consideration and preliminaries

Consider a linear system with two constant delays

$$\dot{x}(t) = Ax(t) + A_1x(t-h_1) + A_2x(t-h_2), \tag{1}$$

where $x(t) \in \Re^n$ is the state; A, A_1 and A_2 are known real constant matrices; the time delays h_1 and h_2 are continuous constant differentiable function satisfying

$$0 \leqslant h_{m1} \leqslant h_1 \leqslant h_{M1}, \quad 0 \leqslant h_{m2} \leqslant h_2 \leqslant h_{M2}, \tag{2}$$

The following two lemmas are required for the derivation of the main result in this paper.

Lemma 1 (Jensen's Inequality [31]). *For any constant matrix* $0 < R$, $0 \leqslant \alpha < \beta$ *and* $0 < \gamma = \beta - \alpha$ *the following bounding holds:*

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \leqslant \gamma^{-1}\begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}. \tag{3}$$

Right Hand Side (RHS) of the above is nonconvex in γ. *An equivalent but convex in* γ *representation of this [31] can be written as:*

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta)R\dot{x}(\theta) \leqslant \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \left\{ \begin{bmatrix} M+M^T & -M+N^T \\ * & -N-N^T \end{bmatrix} + \gamma \begin{bmatrix} M \\ N \end{bmatrix} R^{-1} \begin{bmatrix} M \\ N \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}, \tag{4}$$

where M and N are free weighting matrices of appropriate dimensions.

Lemma 2 (Schur complement [17]). *For given constant matrices* X_1, X_2 *and* X_3 *of appropriate dimensions, where* $X_1^T = X_1$ *and* $X_2^T = X_2$, *then*

$$X_1 + X_3^T X_2^{-1} X_3 < 0,$$

if and only if $\begin{bmatrix} X_1 & X_3^T \\ X_3 & -X_2 \end{bmatrix} < 0$ *or* $\begin{bmatrix} -X_2 & X_3 \\ X_3^T & X_1 \end{bmatrix} < 0$.

To this end, as noted in the introduction section, the analysis for systems with single delay can easily be extended for systems with two delays. Such a result is the following, the proof of which is presented in Appendix A.

Lemma 3. *System (1) is stable if there exist* $P > 0$, $Q_{ij} > 0$, $R_{ik} > 0$, $i = 1,2$ *and* $j = 1,2,\ldots,4$, *and arbitrary matrices* M_{ik}, N_{ik}, $k = 1,2$ *satisfying these LMIs*

$$\begin{bmatrix} \Theta & \bar{h}_1\Phi_l & \bar{h}_2\Phi_m \\ * & -\bar{h}_1R_{12} & 0 \\ * & * & -\bar{h}_2R_{22} \end{bmatrix} < 0, \quad l = 1,2 \quad \text{and} \quad m = 3,4, \tag{5}$$