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The complex variable meshless local Petrov–Galerkin method for elastodynamic problems

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ABSTRACT

The complex variable meshless local Petrov–Galerkin (CVMLPG) method is further developed for structural dynamic analyses of two-dimensional (2D) solids. The complex variable moving least-square (CVMLS) approximation is used to construct the shape function and the Heaviside step function is chosen for representing the test function. In the construction of the well-performed shape function, the trial function of 2D problem is formed with a one-dimensional (1D) basis function, thus improving computational efficiency. The governing elastodynamic equations are transformed into a standard local weak formulation, and then it is discretized into a meshfree system of time-dependent equations, which are solved by the standard implicit Newmark time integration scheme. Numerical results obtained from the proposed CVMLPG method are compared with the exact solutions and those of the conventional MLPG method and excellent agreement is achieved.

1. Introduction

To eliminate the burdensome mesh generations of the traditional numerical method such as the finite element method, increasing attention and efforts have been paid to meshless methods [1–6]. The meshless local Petrov–Galerkin (MLPG) method based on the moving least-square (MLS) approximation is one of recent meshless approach [7]. The main advantage of this method over other meshless methods is that it does not require a mesh either for the approximation of the trial function or for the integration of the weak form. The MLPG method is a "truly meshless" approach and has been demonstrated to be quite successful in solving science and engineering problems [8–12].

Although the MLPG method has been applied to solve many problems, there still exist some inconveniences or disadvantages when using the MLPG method. Because the MLS shape functions lack the Kronecker delta function property, it is not easy to accurately impose essential boundary conditions in the MLPG method. Furthermore, in practical numerical processes, the computational cost of the MLS approximation is quite burdensome. Sometimes this method can form an ill-conditioned system of equations. These disadvantages decrease the computational efficiency of the MLS method.

In order to eliminate this shortcoming of the MLS shape function, a complex variable moving least-square (CVMLS) approximation can be employed instead of the traditional MLS approximation to construct the meshless shape function. Compared with the traditional MLS approximation, the advantages of the CVMLS approximation are that the trial function of a 2D problem is formed with 1D basis function, which leads to the fewer nodes used to form the meshless shape function. Consequently the efficiency of the CVMLS method is improved. Liew [13] and Cheng [14–16] proposed the CVMLS

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approximation and has successfully developed the complex variable element free Galerkin method (CVEFG) for elasticity. Then, they solved fracture [17], elasto-plasticity [18], elastodynamics [19], transient heat conduction [20], and large deformation problems [21]. Later, some meshless methods based on the CVMLS approximation have also been developed and applied to solve a variety of engineering problems, for instance, the complex variable boundary element-free method [22], the complex variable meshless local Petrov–Galerkin(CVMLPG) method [23–25], the complex variable meshless manifold method (CVMMM) [26,27]. On the basis of reproducing kernel particle method and the complex variable theory, complex variable reproducing kernel particle method was developed by Chen and Cheng [28,29].

In the present research, the CVMLPG method is implemented for elastodynamic problems and aims the sensibility and effectiveness of the method. To this end, the CVMLS approximation is described briefly. Next, the discretization procedure and the discrete formulation together with the Newmark method are presented. Finally, numerical examples are presented to verify the validity of the CVMLPG method for elastodynamic problems.

2. Complex variable moving least-square approximation

In the CVMLPG method, the CVMLS approximation is used for the approximation of the field variables. According to the CVMLS [13], the approximation function $u^h(z)$ of the field variables $\tilde{u}(z)$ at an arbitrary point z in the domain Ω is defined by

$$u^h(z) = u_1^h(z) + iu_2^h(z) = \sum_{j=1}^{m} p_j(z)a_j(z) = \boldsymbol{p}^{\mathrm{T}}(z)\boldsymbol{a}(z), \quad (z = x_1 + ix_2 \in \Omega), \tag{1}$$

where $\boldsymbol{p}(z)$ is a complete polynomial basis vector of order m and $\boldsymbol{a}(z)$ is vector of coefficients (to be determined) which is a function of the space coordinate z.

In general, for 2D problems, the basis functions can be chosen as

$$\text{Linear basis} : \boldsymbol{p}(z) = (1, z)^{\mathrm{T}}, \tag{2}$$

$$\text{Quadratic basis} : \boldsymbol{p}(z) = (1, z, z^2)^{\mathrm{T}}. \tag{3}$$

The local approximation at point z is defined as

$$u^h(z, \bar{z}) = \sum_{j=1}^{m} p_j(\bar{z})a_j(z) = \boldsymbol{p}^{\mathrm{T}}(\bar{z})\boldsymbol{a}(z), \tag{4}$$

where $\bar{z}$ is the node in the local support domain of point z.

The coefficient vector $\boldsymbol{a}(z)$ is obtained by minimizing the difference between the local approximation and the function, and it is defined as

$$J = \sum_{I=1}^{n} w(z - z_I)[u^h(z, \bar{z}) - \tilde{u}(z_I)]^2 = \sum_{I=1}^{n} w(z - z_I)\left[\sum_{j=1}^{m} p_j(\bar{z})a_j(z) - \tilde{u}(z_I)\right]^2, \tag{5}$$

where n is the number of nodes in the neighborhood of point z in which the weight function $w(z - z_I) > 0$, and

$$\tilde{u}(z_I) = u_1(z_I) + iu_2(z_I). \tag{6}$$

Describe Eq. (5) in matrix form as

$$J = (\boldsymbol{P}\boldsymbol{a} - \tilde{\boldsymbol{u}})^{\mathrm{T}}\boldsymbol{W}(z)(\boldsymbol{P}\boldsymbol{a} - \tilde{\boldsymbol{u}}), \tag{7}$$

where $\tilde{\boldsymbol{u}}$ is the vector of nodal unknowns, and it can be expressed as

$$\tilde{\boldsymbol{u}} = (\tilde{u}(z_1), \tilde{u}(z_2), \ldots, \tilde{u}(z_n))^{\mathrm{T}} = \boldsymbol{Q}\boldsymbol{u}, \tag{8}$$

with

$$\boldsymbol{Q} = \begin{pmatrix} 1 & i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & i & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & i \end{pmatrix}_{n \times 2n}, \tag{9}$$

$$\boldsymbol{u} = (u_1(z_1), u_2(z_1), u_1(z_2), u_2(z_2), \ldots, u_1(z_n), u_2(z_n))^{\mathrm{T}}, \tag{10}$$