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The relative efficiency of Liu-type estimator in a partially linear model

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ABSTRACT

In this paper, we study the partially linear model, $y = X\beta + f + \varepsilon$. We introduce a new Liu-type estimator in a partially linear model, then we compared the new estimator with the two-step estimator in the mean squared error sense. Finally, we give a simulation study to explain the validity and feasibility of the approach.

1. Introduction

Consider the following partially linear model

$$y_i = x_i'\beta + f(t_i) + \varepsilon_i, \quad i = 1, \ldots, n \tag{1.1}$$

where $y_i's$ are observations, $x_i' = (x_{i1}, \ldots, x_{ip})$ and $x_1, \ldots, x_n$ are known p-dimensional with $p \leqslant n$. $t_i's$ are values of an extra univariate variable such as the time at which the observation is made, $\beta = (\beta_1, \ldots, \beta_p)'$ is an unknown parameter vector. $f(\cdot)$ is an unknown smooth function, and $\varepsilon_i's$ are random error supposed to be *i.i.d.* $N(0, \sigma^2)$ distributed.

Use matrix vector notation, model (1.1) can be written as follows:

$$y = X\beta + f + \varepsilon, \tag{1.2}$$

where $y = (y_1, \ldots, y_n)',\ X' = (x_1, \ldots, x_n),\ f = (f(t_1), \ldots, f(t_n))'$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)'$.

Engle et al. [1] also called model (1.1) as a partial spline model. $f(t)$ has been called as smooth part of the model and suppose that it represents a smooth unparameterized functional relationship. The main problem for us to consider is to estimate unknown parameter vector β and nonparametric function f from the data $\{y_1, x_i, t_i\}$. In this paper, we mainly discuss how to estimate unknown parameter vector β and nonparametric function f. If we know the estimator of β, then we can obtain the estimator of function f.

There are many methods to estimate β and f. such as, penalized least-squares (see [2]), smoothing splines (see [3]), piecewise polynomial (see [4]) and two steps estimation methods (see [5]). Hu [6] used two-step way and introduced a ridge estimators for the partially linear model. Duran et al. [7] also discussed the two-step method. The main thought of two steps estimation is the following: the first step, $f(t, \beta)$ is defined with supposition where β is assumed to be known; the second step, the estimator of parametric β is attained by a least-squares method; Then we may obtain $\hat{f}(t, \hat{\beta})$.

When the regressor exists multicollinearity, many authors improved to overcome this problem. Tabakan and Akdeniz [8] introduced a difference-based ridge estimator in partially linear model. Duran et al. [9] proposed a difference-based ridge

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and Liu estimator in partially linear model. Tabakan [10] consider the semiparametric regression model with linear equality restrictions and proposed a new restricted difference-based ridge estimator.

In this paper, we use two-step method to introduce a new Liu-type estimator in partially linear model. We also discuss the superiority of the new estimator with the two-step estimator in the terms of the mean squared error.

The paper is organized as follows. In Section 2, the Liu-type estimator in partially linear model is introduced. In Section 3, we give the comparison of the new estimator and the two-step estimator in the mean squared error sense and we give a method to choose the biasing parameter in Section 4. A simulation study is given to illustrate the new method in Section 5 and some conclusion remarks are given in Section 6.

2. The new estimator

In the following, we introduce a Liu-type estimation method based on a two steps estimation process. In the first step, we suppose that β is known, and then the nonparametric estimator of f is given as follows:

$$f(t,\beta) = S(y - X\beta), \tag{2.1}$$

where $S = (I + \alpha K)^{-1}$ is a smoother matrix which depends on a smoothing parameter α and K is a symmetric nonnegative matrix [2]. Then based on $\{y_i - x_i'\beta, t_i\}$ $(i = 1,\ldots,n)$, and $S = S(t_1,\ldots,t_n)$ is an $n \times n$ positive-definite smoother matrix from univariate cubic spline smoothing [7]. Consider the following model

$$\tilde{y} = \tilde{X}\beta + \tilde{\varepsilon}, \tag{2.2}$$

where $\tilde{y} = (I-S)y,\ \tilde{X} = (I-S)X,\ \tilde{f} = (I-S)f,\ \varepsilon^* = (I-S)\varepsilon$ and $\tilde{\varepsilon} = \tilde{f} + \varepsilon^*$. (2.2) is linear model.

Least square estimator of β of semiparametric regression model is got by minimizing:

$$(\tilde{y} - \tilde{X}\beta)'(\tilde{y} - \tilde{X}\beta). \tag{2.3}$$

If we suppose that $\tilde{X} = (I-S)X$ has full column rank, then we obtain

$$\hat{\beta}_p = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}, \tag{2.4}$$

$$\hat{f}_p = S(y - X\hat{\beta}_p). \tag{2.5}$$

The estimator can also be called as two-step estimator $\hat{\beta}_p = \hat{\beta}_{TS}$. In the second step, we add a penalizing function $\|\frac{d\hat{\beta}_p}{k^{1/2}} - k^{1/2}\beta\|^2$ to the least squares objective (2.3) is same to minimizing the criterion

$$(\tilde{y} - \tilde{X}\beta)'(\tilde{y} - \tilde{X}\beta) + \left(\frac{d\hat{\beta}_p}{k^{1/2}} - k^{1/2}\beta\right)'\left(\frac{d\hat{\beta}_p}{k^{1/2}} - k^{1/2}\beta\right), \tag{2.6}$$

we obtain its solution, namely

$$\hat{\beta}_p(k,d) = (\tilde{X}'\tilde{X} + kI)^{-1}(\tilde{X}'\tilde{X} + dI)\hat{\beta}_p, \quad 0 < d < 1,\ d \leqslant k, \tag{2.7}$$

where d and k are tuning parameters.

Then we obtain

$$\hat{f}_p(k,d) = S(y - X\hat{\beta}_p(k,d)). \tag{2.8}$$

Because there is a formal resemblance between (2.7) and the Liu-type estimator of the linear model, we call it a Liu-type estimator of the partially linear model.

Remark 1. Let $k = d$, and suppose that $\tilde{X}'\tilde{X}$ is of full rank, then the Liu-type estimator becomes the two-step estimator of a partially linear model.

Remark 2. Let $f(t) = 0$, then the Liu-type estimator becomes Liu-type estimator in a linear model.

3. Comparison of Liu-type estimator $\hat{\beta}_p(k,d)$ with two-step estimator $\hat{\beta}_{TS}$

In this section, we will give the comparison of the Liu-type estimator $\hat{\beta}_p(k,d)$ with two-step estimator $\hat{\beta}_p$ in the mean squared error (MSE) sense.

Theorem 1. *Suppose* $rank(X) = p$, *and suppose that there exists a matrix* S *such that* $rank[(I-S)X] = rank(\tilde{X}) = p$, *then:*

(a) *for fixed* $k > 0$*:*