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Analysis of a spatial epidemic model with saturated incidence rate

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ABSTRACT

Epidemic models can be used to describe the process of epidemic spreading and provide some information on disease control. In this paper, we investigated a spatial epidemic model with saturated incidence rate. We obtained some qualitative behavior of the epidemic model, including dissipation, persistence, stability and so on, which well enrich the theory of spatial epidemic models.

1. Introduction

In mathematical modeling of disease transmission, Kermack and McKendrick posed a classical epidemic model in 1927 [6]. The total population at t moment is divided into three groups: susceptible, infectious and recovered, where $S(t)$ is the number of susceptible, $I(t)$ is the number of infectious, $R(t)$ is the number of the recovered. They assumed that the contact rate between susceptible and infectious was proportional to the number of the population. Obviously, this assumption was not reasonable.

In 1973, after studying cholera occurred in Bari of Italy, Capasso and Serio introduced a saturation incidence $g(I)S$ into the epidemic models [2], which described the contact of infectious individuals and susceptible individuals, and

$$g(I) = \frac{kI}{1 + \alpha I},$$

where kI represents infectivity, $1/(1 + \alpha I)$ is inhibitions from changes of behaviors of susceptible individuals as the number of susceptible increases. When the number of infectious increases, $g(I)$ tends to a saturated level.

Nonlinear incidence rate

$$h(I) = \frac{kI^2}{1 + \alpha I^2}.$$

was also considered in epidemic models [10], and rich dynamics were obtained including a limit cycle, two limit cycles, homo-clinic cycles and so on. However, one can find that

$$g'(I) = \frac{k(1 + \alpha I) - \alpha kI}{(1 + \alpha I)^2} = \frac{k}{(1 + \alpha I)^2} > 0.$$

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and

$$h'(I) = \frac{kpI(1+\alpha I^2) - \alpha kqI^3}{(1+\alpha I^2)^2} = \frac{kpI}{(1+\alpha I^2)^2} > 0.$$

That is to say that these incidences are all monotonic, which means that the contact rate of infectious individual and susceptible individual increases with the number of infectious individual increases.

In fact, this situation is not in line with the actual case. For such reason, Xiao and Ruan introduced non-monotone incidence $kI/(1+\alpha I^2)$ to the epidemic models [23]. When the number of infectious gradually increased, it described the psychological effect of certain serious disease on the community. They showed that either the number of infective individuals tends to zero as time evolves or the disease persists. However, the epidemic model can not undergo any bifurcations. In Ref. [24], a non-monotone incidence rate $kI/(1+\beta I+\alpha I^2)$ was introduced to an epidemic model and obtained rich bifurcation behaviors.

From the biological point of view, individual organisms are distributed in space and typically interact with the physical environment and other organisms in their spatial neighborhood [1,17,18,21]. The diffusion of the population is aim to look for food and escape from high risk of disease [16]. For the first case, the individual diffuses towards the direction of the low population density to obtain more abundant resource. And for the second case, the individual will go along the gradient of the infected individuals to avoid infectious with higher infection rate [8].

In Ref. [9], basing on the analysis of the characteristic equation and Lyapunov function, the authors discussed the local and global stability of the endemic equilibrium in an epidemic model. For the corresponding reaction–diffusion model of infectious diseases, they obtained the condition of global asymptotical stability of the endemic equilibrium. Moreover, the stochastic model was studied in detail. However, there is little knowledge on the dynamics of infectious disease models with non-monotone saturated incidence. As a result, we want to investigate the dynamics of a spatial epidemic model and find the differences between non-monotone saturated incidence and monotone saturated incidence.

In this paper, we will present a spatial epidemic model with non-monotonic incidence of saturated mass action and investigate its dynamical behavior. In Section 2, we establish an epidemic model and give the biological meanings of parameters. In Section 3, we show dissipation, persistence and stability of the spatial epidemic models. Finally, some conclusions are given in the last section.

2. An epidemic model

Firstly we consider an epidemic model which is as follows:

$$\frac{dS}{dt} = b - dS - f(I)S + \delta R, \tag{1a}$$

$$\frac{dI}{dt} = f(I)S - (d+\mu)I, \tag{1b}$$

$$\frac{dR}{dt} = \mu I - (d+\delta)R, \tag{1c}$$

where $S(t),\ I(t),\ R(t)$ denote the number of the susceptible, infected, recovered at time t, respectively. b is the addition rate of the population, d is the natural mortality, μ is the natural recovery rate of infected individuals, and δ is recovered rate. Here, $f(I) = kI/(1+\beta I+\alpha I^2)$, where k is proportional constant and α is positive constant. For all $I \geqslant 0,\ \beta$ makes the formula $1+\beta I+\alpha I^2 > 0$ established which needs $\beta > -2\sqrt{\alpha}$.

From the biological point of view, we are interested in dynamical behaviors of the system (1) in the region R^3 of the first quadrant. We first have the conclusion that $S+I+R = b/d$ is the invariant manifold of system (1). As a result, system (1) is equivalent to the following system:

$$\frac{dI}{dt} = f(I)\left(\frac{b}{d} - I - R\right) - (d+\mu)I, \tag{2a}$$

$$\frac{dR}{dt} = \mu I - (d+\delta)R. \tag{2b}$$

System (1) has disease free equilibrium and endemic equilibrium if and only if the system (2) has a disease-free equilibrium (0,0) and endemic equilibrium. Obviously, system (1) always has a disease-free equilibrium $E_0 = \left(\frac{b}{b}, 0, 0\right)$ for all the values of the parameters. In order to find the endemic equilibrium of system (2), we discuss the existence of the positive equilibrium of the system (2).

For simplicity on symbols, we let

$$x = \frac{kI}{d+\delta}, \quad y = \frac{kR}{d+\delta}, \quad \tau = (d+\delta)t$$