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On the truly nonlinear oscillator with positive and negative damping

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ABSTRACT

In this paper the oscillator with a restoring force of rational order of nonlinearity (named truly nonlinear oscillators) and damping (positive and negative) is considered. The mathematical model of the oscillator is a second order differential equation with nonlinear terms of integer or noninteger and also linear and nonlinear damping terms. The approximate solution of the generating differential equation (only the nonlinear deflection and linear viscous damping terms exist) in the form of the Ateb function is obtained using the harmonic balance method. Based on the generating solution the vibrations of the oscillator with damping are obtained by extending the method of time variable amplitude, frequency and phase. The special attention is given to the truly nonlinear damped-van der Pol oscillator. The interaction of the viscous and van der Pol damping on the motion of the truly nonlinear oscillator is investigated. The boundary for the limit cycle motion depending on the order of nonlinearity is analyzed. The interactive influence of the damping coefficients (positive and negative) and the order of nonlinearity on the frequency and the period of the vibration is also analyzed. Two numerical examples are considered. The approximate analytical solutions are compared with numerically calculated ones. The obtained results are in a good agreement.

1. Introduction

Based on the experimental data a significant number of practical devices (like micro-electro-mechanical and nano-electro-mechanical devices, nanometer switches, vibration-, impact- and acoustic isolators, snap-through mechanisms, etc.), but also phenomena in structural mechanics, nanotechnology, chemistry and physics, are modeled as truly nonlinear oscillators with strong nonlinear stiffness and damping properties (see for example [1–3])

$$\ddot{x} + c_\alpha^2 x|x|^{\alpha-1} + f(x,\dot{x}) = 0, \tag{1}$$

where c_α^2 is a constant stiffness coefficient, $\alpha \neq 1 \wedge \alpha \in \mathbb{R}_+$ is the order of stiffness nonlinearity (α is a rational number: integer or non-integer) and $f(x,\dot{x})$ is the function which includes the dissipation in the system. Unfortunately, the analytical solving of such a strong nonlinear differential equation is not an easy task. In the most of the papers dealing with the problem the model is simplified and some special cases of (1) are considered. Thus, the most of oscillators are assumed to be linear ($\alpha = 1$) and the additional function f is assumed to be multiplied with a small parameter $\varepsilon \ll 1$. Numerous analytical solving procedures are developed for such linear differential equations with small nonlinearity (see for example [4]).

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During the last thirty years the investigations are extended to dynamics of the strong nonlinear oscillators of Duffing type (see [5] and the reference list in it) and a large attention is given to the influence of the small viscous damping, which is linearly proportional to the velocity, on the dynamics of the oscillator with cubic nonlinearity. According to the requirements of the real physical system and engineering application also the influence of the quadratic damping is assumed [6–8]. Siewe et al. [9] extended the investigation to the so called Rayleigh–Duffing oscillator with cubic and linear damping terms. The role of nonlinear damping in soft Duffing oscillator with a simultaneous presence of viscous damping has been discussed in [10,11]. Baltanas et al. [12] have studied the effect of including a nonlinear damping term, proportional to the power of velocity, in the dynamics of double-well Duffing oscillator. In [13,14] the Duffing oscillator with positive linear and cubic term and small linear and cubic damping are considered. Recently, Kanai and Yabuno [15] gives the qualitative analyses of the Duffing oscillator with small positive linear and cubic damping and also of the Duffing oscillator driven with small van der Pol damping. In [16] the oscillator with nonlinearity of integer order is analyzed. Approximate solution of the second order differential equation with nonlinearity of polynomial type is assumed in the form of a trigonometric function which is suitable for qualitative, but not also quantitative analyses of the problem. Besides, it is known that the force–deflection relationship and also the damping are very often necessary to be modeled as nonlinear functions of noninteger order. Kovacic [17] gives the qualitative and quantitative analysis of a linear oscillator with additional small nonlinear stiffness and damping terms of any noninteger order.

The intention of the paper is to generalize the previous investigation and to consider the model of a linear damped truly nonlinear oscillator (force–deflection function is of any rational order α) with additional small nonlinear damping $\varepsilon f(x,\dot{x})$

$$\ddot{x}+\kappa\dot{x}+c_\alpha^2 x|x|^{\alpha-1}=\varepsilon f(x,\dot{x}), \tag{2}$$

where κ is the coefficient of the viscous damping. It is of special interest to analyze the motion of the damped truly nonlinear oscillator with additional negative damping of the van der Pol type

$$\ddot{x}+\kappa\dot{x}+c_\alpha^2 x|x|^{\alpha-1}=\varepsilon\dot{x}(1-|x|^2). \tag{3}$$

The investigation published in this paper involved:

1. Determination of the approximate solution of the truly nonlinear oscillator of any order of nonlinearity (of integer or non-integer type) with significant viscous damping

$$\ddot{x}+\kappa\dot{x}+c_\alpha^2 x\,|x|^{\alpha-1}=0 \tag{4}$$

and the proof of the correctness of that solution.
2. Generalization of the solution procedure for the linear damped truly nonlinear oscillator with additional nonlinear terms (2).
3. Investigation of the linear damped truly nonlinear-van der Pol oscillator (3) and determination of the limits for cyclic motion.

The results given in this paper are organized in six sections. In Section 2, using the generalized harmonic balance method an approximate solution of the generating Eq. (4), in the form of the special Ateb function [18], is determined. The accuracy of the generating solution is proved. In Section 3 we developed an analytical averaging method for solving of the differential equation with small added terms (2). The solution procedure is based on the perturbation of the generating solution with time variable amplitude, frequency and phase. In Section 4 the motion of the oscillator with van der Pol damping is considered. Analyzing the approximate solution for truly nonlinear and damped-van der Pol oscillator (3), the steady-state amplitude of the limit cycle motion is obtained. Two numerical examples are considered: the linear damped Duffing–van der Pol oscillator and a damped oscillator with nonlinearity of order 1/3-van der Pol oscillator. The analytically obtained results are compared with numerical ones. The paper ends with Conclusion (Section 5). In Appendix A a short introduction to the Ateb functions and their averaging is given.

2. Truly nonlinear oscillator with linear viscous damping

We derived the approximate solution for the viscous damped truly nonlinear oscillator of any rational order (4) using the generalized harmonic balance method. Namely, we assumed the solution of (4) in the form

$$x=A\exp(-\delta t)ca(\alpha,1,\psi(t)) \tag{5}$$

with

$$\psi(t)=\theta+\int\omega(t)dt, \tag{6}$$

where ca is the cosine Ateb function (see Appendix A) which depends on the order of nonlinearity α and the unknown time function $\psi(t)$ which satisfies the relation (6), δ is an unknown constant value, A and θ are arbitrary constants and $\omega(t)$ is an unknown time function which has to be determined. Using the derivatives of the Ateb functions [18]