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Bifurcation mechanism of bursting oscillations in parametrically excited dynamical system ☆

Qinsheng Bi *, Ran Zhang, Zhengdi Zhang

Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, PR China

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ABSTRACT

The evolution of bursting oscillations in a parametrically excited dynamical system with order gap between the excited frequency and the natural frequency is investigated in this paper. By regarding the periodic excited term as a slow-varying parameter, different forms of bifurcations of the system are obtained. Base on the overlap between the bifurcation diagram and the phase portrait, the mechanism of different types of bursting oscillations are obtained. Furthermore, some phenomena in bursting oscillations such as symmetry breaking behavior are explained through the bifurcations occurring at the transitions between the quiescent state (QS) and spiking state (SP).

1. Introduction

Many dynamical systems in physics, chemistry, biology and geo-physics involve multiple time scales [1,2], which often behave in periodic bursting oscillations characterized by a combination of relatively large amplitude and nearly harmonic small amplitude oscillations, conventionally denoted by N^K with N and K corresponding to large and small amplitude oscillations, respectively [3]. Bursting phenomena can be observed when the variables alternate between two states, in which the quiescent state (QS) corresponds to the stage when all the variables are at rest or exhibit small amplitude oscillations, while the spiking state (SP) corresponds to the stage when the variables behave in large amplitude oscillations [4]. At the transitions between QS and SP, two important bifurcations can be found, i.e., bifurcation of a quiescent state that leads to repetitive spiking and bifurcation of a spiking attractor that leads to quiescence [5].

Many results related to the effect of two time scales are presented, such as the symmetric bursting behaviors in the generalized FitzHugh-Nagumo model [6], cusp type bursting in photosensitive B-Z reaction [7], tea-cup attractor in ecological model[8] and non-smooth bifurcation in the bursting oscillations for neuron models [9]. Up to now, most of the reports are focused on the autonomous systems with obvious slow and fast subsystems [10], while for the non-autonomous systems such as periodically excited oscillators, when there exists order gap between the excited frequency and natural frequency, the dynamics may behave in relaxation oscillations since both two frequencies may be observed in the time-series [11]. No obvious slow and fast subsystems can be defined, which may lead to the complication of analysis of QS and SP as well as the related bifurcation forms [12]. Furthermore, the slow-fast analysis method [13] can not be directly employed to explain the mechanism of the bursting, resulting in the problem how to explore characteristics of the bursting behaviors in non-autonomous systems [14].

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* Corresponding author.
E-mail address: qbi@ujs.edu.cn (Q. Bi).

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In this paper, we consider a typical parametrically excited self-exciting dynamo model [15] and take suitable parameter values so that order gap exists between the excited frequency and the natural frequency to investigate the evolution of the dynamics with special forms of two time scales, especially the bursting oscillations as well as the related mechanism.

2. Mathematical model

A three dimensional system of nonlinear ordinary differential equations has been established to describe the dynamics of the self-exciting dynamo action in which a Faraday disk and coil are arranged in series with either capacitor or a motor [15]. The non-dimensional system governing the dynamo with periodic parametric excitation acting on one of the state variables to control the behaviors can be written in the form

$$\dot{x} = x(y-1) - \beta z, \quad \dot{y} = \alpha(1-x^2) - \kappa y, \quad \dot{z} = x - (\rho + w)z, \tag{1}$$

where $w = A\sin(\Omega t)$, represents the periodic parametric excitation with amplitude A and frequency Ω. Bifurcation properties of (1) without the excitation have been investigated in details [14], in which the Hopf bifurcation may lead to the periodic oscillations according to the natural frequency Ω_N. Generally, when the periodic excitation is applied to the oscillator, the dynamics may behave in 2-D torus with two frequencies Ω_N and Ω for the case without resonance. However, when order gap exists between Ω_N and Ω, for example, $\Omega \ll \Omega_N$, the effect of two time scales described by $\Omega_N t$ and Ωt appears, which may lead to different types of bursting oscillations with the variation of parameters.

3. Bifurcation analysis

When there exists order gap between the excited frequency and the natural frequency, the effect of two time scales may evolve in the system, which often behaves in bursting oscillations. Here we fix the excited frequency at $\Omega = 0.05$, which is far smaller than the natural frequency Ω_N, to investigate dynamics of the vector field.

Three equilibrium points for system (1) can be found, expressed by $EQ_0(0, \alpha/\kappa, 0)$ and $EQ_\pm(\pm\sqrt{\Delta}, (1-\Delta)\alpha/\kappa, \pm\sqrt{\Delta}/\rho)$, respectively, where $\Delta = [\alpha(\rho+w) - \kappa(\beta+\rho+w)]/[\alpha(\rho+w)]$, the stabilities of which can be determined using the associated characteristic equations, expressed by

$$(\lambda+\kappa)\left[\kappa\lambda^2 + (\kappa\rho + \kappa w + \kappa - \alpha)\lambda + \kappa\rho + \kappa w + \kappa\beta - \alpha\rho - \alpha w\right]/\kappa = 0, \tag{2}$$

for EQ_0 and

$$F_{\lambda\pm} = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3, \tag{3}$$

for $EQ_\pm$, where

$$\begin{aligned} a_1 &= (\rho + w + \kappa + 1) + \alpha(\Delta - 1)/\kappa, \\ a_2 &= (\beta + \kappa + \rho + w + 3\alpha\Delta - \alpha + \rho\kappa + w\kappa) + \alpha(\rho + w)(\Delta - 1)/\kappa, \\ a_3 &= \kappa(\rho + w + \beta) + \alpha(\rho + w)(3\Delta - 1). \end{aligned} \tag{4}$$

Therefore, EQ_0 is stable for $\kappa\rho + \kappa w + \kappa - \alpha > 0$ and $\Delta < 0$, while both $EQ_\pm$ are stable for $a_1a_2 - a_3 > 0,\ a_3 > 0$ and $a_1 > 0$. When the stability conditions are violated, different types of bifurcations may occur.

Pitchfork bifurcation. For $\Delta = 0$, the three equilibrium points may join together to form a cusp point, the eigenvalues of the characteristic equation at which can be expressed by $\lambda_1 = 0,\ \lambda_2 = -\kappa$ and $\lambda_3 = \alpha/\kappa - (\rho + w - 1)$, implying pitchfork bifurcation may occur at

$$\mathbf{PF}: \ \kappa(\rho + w + \beta) - \alpha(\rho + w) = 0, \tag{5}$$

corresponding to the colliding between the two symmetric nontrivial equilibrium and the trivial one.

Hopf bifurcation. A pair of pure imaginary eigenvalues, denoted by $\Omega_{H1}^2 = (\rho + w + \beta) - \alpha(\rho + w)/\kappa$, related to the Eq. (2) can be obtained for

$$\mathbf{HB_1}: \ \kappa(\rho + w + 1) - \alpha = 0, \tag{6}$$

at which Hopf bifurcation associated with EQ_0 may occur, leading to periodic oscillations, while for

$$\mathbf{HB_2}: \ a_1a_2 - a_3 = 0, \quad (a_1 > 0,\ a_3 > 0), \tag{7}$$

Hopf bifurcations may take place, leading to possible periodic oscillations surrounding EQ_+ or EQ_-, respectively.

Remarks

① The dynamical behaviors associated with the bifurcations can be demonstrated through numerical simulation by taking w as a real parameter.