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Laguerre collocation method for the flow and heat transfer due to a permeable stretching surface embedded in a porous medium with a second order slip and viscous dissipation

M.M. Khader *

Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh, Saudi Arabia

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ABSTRACT

A numerical method is given for studying the effect of viscous dissipation on the steady flow with heat transfer of Newtonian fluid towards a permeable stretching surface embedded in a porous medium with a second order slip. The governing nonlinear partial differential equations are converted into nonlinear ordinary differential equations by using similarity variables. The proposed method is based on replacement of the unknown function by truncated series of well-known Laguerre expansion of functions. An approximate formula of the derivative is introduced. Special attention is given to study the convergence analysis and derive an upper bound of the error of the presented approximate formula. The introduced method converts the proposed equation by means of collocation points to a system of algebraic equations with Laguerre coefficients. Thus, by solving this system of equations, the Laguerre coefficients are obtained. Graphically results are shown for non-dimensional velocities and temperature. The effects of the porous parameter, the suction (injection) parameter, Eckert number, first and second order velocity slip parameter and the Prandtl number on the flow and temperature profiles are presented. Moreover, the local skin-friction and Nusselt numbers are presented. A comparison of numerical results is made with the earlier published results under limiting cases.

1. Introduction

Due to the widespread applications in industry and manufacturing processes, such as, polymer extrusion, wire and fiber coating, foodstuff processing, paper production, hot rolling, solidification of liquid crystals, petroleum production, continuous cooling and fibers spinning, exotic lubricants and suspension solutions, the boundary layer flow past a stretching surface has attracted considerable attention of researchers during the past few decades. Much work on the boundary-layer Newtonian fluids has been carried out both experimentally and theoretically. Crane [1] was the first one who obtained an elegant analytical solution to the boundary layer equations for the steady two-dimensional flow due to a stretching surface in a quiescent incompressible fluid taking into account the case of a linearly stretched surface. The problem of Newtonian flow past a stretching surface has been extended by many authors [2–10] in various ways.

In all the above-mentioned studies the slip velocity effect is negligible. However, the slip velocity effect is interesting macroscopically physical phenomenon in fluid mechanics. Experimental results [11] have shown that, the empirical non-slip

* Permanent address: Department of Mathematics, Faculty of Science, Benha University, Benha, Egypt.
E-mail address: mohamedmbd@yahoo.com

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boundary condition may break down depending on the fluid properties and interfacial roughness. Therefore, the slip effect should be taken into account which leads to the requirement of a slip boundary condition. The earliest slip boundary condition was proposed by Navier [12]. He showed a linear relationship between the slip velocity and the shear rate at the wall. But according to the results from molecular dynamics simulations, Thompson and Troian [13] discovered that the slip velocity is related to the slip length, the shear rate at the wall and a critical shear rate at which the slip length diverges. Wall slip readily occurs for an array of complex fluid such as emulsions, suspensions, foams, and polymer solutions. Also, the fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. Turkyilmazoglu [14] investigated the MHD slip flow of an electrically conducting, viscoelastic fluid past a stretching surface. Recently, Megahed [15] investigated the numerical solution for the slip velocity and variable viscosity effects on the flow and heat transfer of a non-Newtonian power-law fluid over a stretching surface in the presence of thermal radiation and constant heat flux.

The prediction of heat transfer characteristics for the Newtonian fluids in porous media is very important due to its practical engineering applications, such as oil recovery, geothermal energy recovery, ground water pollution, thermal energy storage, flow through filtering media and food processing. So, the purpose of the present paper is to investigate the numerical solution for the viscous dissipation and second order slip effects on the flow and heat transfer of a Newtonian fluid over a permeable stretching surface embedded in porous medium.

Spectral collocation methods are efficient and highly accurate techniques for numerical solution of non-linear differential equations. The basic idea of the spectral collocation method is to assume that the unknown solution $y(x)$ can be approximated by a linear combination of some basis functions, called the trial functions, such as orthogonal polynomials. The orthogonal polynomials can be chosen according to their special properties, which make them particularly suitable for a problem under consideration. In [16], Khader introduced an efficient numerical method for solving the fractional diffusion equation using the shifted Chebyshev polynomials. In [17] the generalized Laguerre polynomials were used to compute a spectral solution of a non-linear boundary value problems. The generalized Laguerre polynomials constitute a complete orthogonal sets of functions on the semi-infinite interval $[0,\infty)$. Convolution structures of Laguerre polynomials were presented in [18]. Also, other spectral methods based on other orthogonal polynomials are used to obtain spectral solutions on unbounded intervals [19].

The spectral collocation method is used to solve many problems, in more papers such as [20–22]. In this work, we use the properties of the Laguerre polynomials to derive an approximate formula of the derivative $D^{(n)}y(x)$ and estimate an error upper bound of this formula, then we use this formula to solve numerically the proposed problem.

2. Formulation of the problem

Consider a steady, two-dimensional boundary layer flow of an incompressible Newtonian fluid over a permeable stretching sheet embedded in a porous medium with viscous dissipation and second order slip effects. The origin is located at a slit, through which the sheet (see Fig. 1) is drawn through the fluid medium. The x-axis is chosen along the sheet and y-axis is taken normal to it. The continuous stretching sheet is assumed to have the velocity $U = cx$ where x is the coordinate measured along the stretching surface and $c(>0)$ is a constant for a stretching sheet and temperature distribution for the sheet is assumed to take the form $T_w = T_\infty + Ax^r$ where T_w is the temperature of the sheet, T_∞ is the temperature of the fluid far away from the sheet, A and r are constants. Also, the sheet is assumed to be porous and the suction (injection) velocity $v_w > 0$ ($v_w < 0$) is taken into consideration. Likewise, the fluid properties are assumed to be constants. Further, the slip velocity at the sheet, is assumed to become [23–25]

$$u_{slip} = a\left(\frac{\partial u}{\partial y}\right) + b\left(\frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

where a and b are constants.

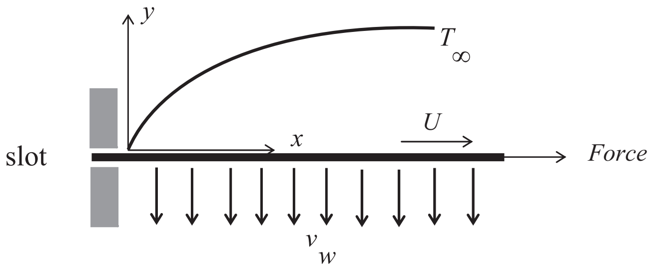

Fig. 1. Flow geometry and coordinate system.