Applied Mathematics and Computation 243 (2014) 559–569

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A method of particular solutions for multi-point boundary value problems

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ARTICLE INFO

Keywords:
Linear problems
Nonlinear problems
Multi-point boundary value problem
Particular solutions
Numerical solution

ABSTRACT

The paper presents a new semi-analytic numerical method for solving multi-point boundary value problems with linear and nonlinear equations of the second and higher orders. The method is based on the use of the particular solutions of the linearized equation. Presented in the paper examples include the solution of equations with differential operators of the second, third and fourth orders.

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1. Introduction

In this paper, we consider the following multi-point boundary value problems (MPBVPs):

$$u^{(s)} = F\left(u, u^{(1)}, \dots u^{(s-1)}, x\right), \quad x \in [0,1], \tag{1}$$

$$\sum_{j=0}^{s-1} a_{j,k} u^{(j)}\left(\xi_{j,k}\right) = d_k, \quad 0 \leqslant \xi_{j,k} \leqslant 1, \; k = 1, \dots, s. \tag{2}$$

Some of the coefficients $a_{j,k}, \; d_k$ could be equal to zero. Sometimes we write the equation in the form

$$u^{(s)} = F\left(u, u^{(1)}, \dots u^{(s-1)}, x\right) + f(x) \tag{3}$$

highlighting the part $f(x)$ that does not depend on u.

The linear analogs of (3)

$$u^{(s)} = \sum_{k=0}^{s-1} A_k(x) u^{(k)}(x) + f(x), \quad x \in [0,1] \tag{4}$$

are also considered in the paper.

We assume that $F, \; A_k$ and f are smooth enough functions of each arguments. Such problems often arise in many branches of applied science. For instance, Hajji in [7] considered MBVP which occurs in many areas of engineering applications. For instance in modelling the flow of fluid such as water, oil and gas through ground layers, where each layer constitutes a subdomain. Also as it is said in [4] large size bridges are sometimes contrived with multi-point supports which correspond to a multi-point boundary value condition. This application of the MBVP is also considered in [13].

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http://dx.doi.org/10.1016/j.amc.2014.06.014
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The shooting method is used to solve multi-point boundary value problems in [8,19]. Geng [5] proposed a method for a class of second-order three-point BVPs by converting the original problem into an equivalent integro-differential equation. Lin and Lin [9] introduced an algorithm for solving a class of multi-point BVPs by constructing a reproducing kernel satisfying the multi-point boundary conditions. The analytical approximate solutions of the third order three-point boundary value problems using reproducing kernel method are investigated in [18]. The combination of the reproducing kernel method and an iterative technique for solving second-order nonlinear MBVPs is presented in [6,10]. The Adomian decomposition method [14], the homotopy analysis method [2], the optimal homotopy asymptotic method [1] and He's variational iteration method [15] are also employed to solve MBVPs.

In this paper we use the semi-analytic method proposed earlier in [11,12] to solve nonlinear two-point BVPs. This method is described in detail in the next section. The numerical examples illustrating the application of the method to solving MBVPs with equations of the second, third and fourth orders are placed in Section 3. Section 4 ends this paper with a brief conclusion.

2. Main algorithm

The method presented is as follows. Let $\varphi_m(x)$ be some system of basis functions on $[0,1]$. Here we consider the following two ones:

(1) the monomials:

$$\varphi_m(x) = x^{m-1}, \quad m = 1,2\ldots,M. \tag{5}$$

(2) the trigonometric basis functions:

$$\varphi_m(x) = \sin(0.5m\pi(x+0.5)), \quad m = 1,2\ldots,M. \tag{6}$$

The particular solutions of the equation

$$\phi_m^{(s)}(x) = \varphi_m(x), \tag{7}$$

which correspond to the basis functions (5) are:

$$\phi_m(x) = \frac{x^{m+s-1}}{m(m+1)\ldots(m+s-1)}. \tag{8}$$

When the trigonometric basis functions (6) are used, the corresponding ϕ_m can be found in the form:

$$\phi_m(x) = (-1)^k \frac{2^s \sin(0.5m\pi(x+0.5))}{(\pi m)^s} \text{ — for even } s = 2k \tag{9}$$

and

$$\phi_m(x) = (-1)^k \frac{2^s \cos(0.5m\pi(x+0.5))}{(\pi m)^s} \text{ — for odd } s = 2k+1. \tag{10}$$

We denote

$$\Phi_m(x) = \phi_m(x) + c_{m,0} + c_{m,1}x + \cdots + c_{m,s-1}x^{s-1}. \tag{11}$$

So, $\Phi_m^{(s)}$ satisfy the same Eq. (7) as ϕ_m:

$$\Phi_m^{(s)}(x) = \phi_m^{(s)}(x) = \varphi_m(x). \tag{12}$$

The free coefficients $c_{m,i}$ in (11) are chosen in such a way that Φ_m satisfy the homogeneous boundary conditions (2):

$$\sum_{j=0}^{s-1} a_{j,k}\Phi_m^{(j)}(\xi_{j,k}) = 0, \quad k = 1,\ldots,s. \tag{13}$$

Substituting (11) in (13), one gets a linear system for $c_{m,0}, c_{m,1}\ldots c_{m,s-1}$.

We assume that the nonlinear term in (3) can be approximated by the linear combinations of the basis functions $\varphi_m(x)$:

$$F(u, u^{(1)}, \ldots, u^{(s-1)}, x) = \sum_{m=1}^{M} q_m\varphi_m(x). \tag{14}$$

Substituting this approximation in the initial equation (3), one gets

$$u_M^{(s)}(x) = \sum_{m=1}^{M} q_m\varphi_m(x) + f(x). \tag{15}$$

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