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Swarm/flock optimization algorithms as continuous dynamic systems

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ABSTRACT

A new general typology of optimization algorithms, inspired to classical swarm intelligence, is presented. They are obtained by translating the numerical swarm/flock-based algorithms into differential equations in the time domain and employing analytical closed-forms written in the continuum. The use of circulant matrices for the representation of the connections among elements of the flock allowed us to analytically integrate the differential equations by means of a time-windowing approach. The result of this integration provides functions of time that are closed-forms, suitable for describing the trajectories of the flock members: they are directly used to update the position and the velocity of each bird/particle at each step (time window) and consequently they substitute in the continuous algorithm the classical updating rules of the numerical algorithms. Thanks to the closed forms it is also possible to analyze the effects due to the tuning of parameters in terms of exploration or exploitation capabilities. In this way we are able to govern the behavior of the continuous algorithm by means of non stochastic tuning of parameters. The proposed continuous algorithms have been validated on famous benchmark functions, comparing the obtained results with the ones coming from the corresponding numerical algorithms.

1. Introduction

The Swarm Intelligence applied to optimization was first introduced in [1] by James Kennedy and Russell Eberhart in 1995. They presented the Particle Swarm Optimization (PSO), a heuristic inspired to the social and collective behavior shown by several animal species such as flock of birds or school of fishes. PSO starts from Reynolds's works [2] for computer graphics and from Heppner and Grenander [3]. Afterwards, Kennedy and Eberhart [1] used a one-to-one correspondence between the motion of a flock searching food and the iterative steps of an algorithm searching the best solution for optimization. However in [1] some flock behavior has been neglected [2], for example, the nearest neighbor-velocity-matching was removed, and so on. Although these changes were useful for simplifying the implementation and the speed of the algorithm, they altered the virtual collective movement of the flock. Thus, the algorithm presented in [1] seems to be more adequate to simulate a swarm of insects rather than a flock of birds. This justifies the choice to call it PSO. Ever since, many authors have published a lot of works about the PSO applied on several fields of science, such as engineering, physics, chemistry, artificial intelligence, economics, and so on, testifying the PSO success in many scientific communities (e.g. see [4–9] and the

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references within). Obviously, a large series of changes from the original PSO have been proposed in order to improve its performances. In particular, many works focused on the way to manage the tuning of parameters for achieving better convergence to the global optimum and/or for improving exploration capability for multimodal problems (e.g. see [10–19] and the reference within): in this context, in literature several attempts have been made to treat the swarm-based numerical algorithms as continuous dynamical systems [20–23] with the aim to investigate on the stability of PSO and the effect of the tuning of parameters on the optimization performance. Further, the topological rules on PSO performances have been discussed [24–30]. The basic idea of topological rules is to link each member of the swarm with others by generating more complex information exchanging among particles than the simple use of the Global best. A particular way to link the particle behaviors is achieved by using a kind of neighbor-velocity-matching. The algorithm thus obtained, called Flock of Starlings Optimization (FSO) [31,32] is inspired to a naturalistic work presented in [33], where the authors show the results obtained by studying the collective behavior of a flock of starlings (common European little birds). In particular, FSO uses topology for exchanging the information about the current velocity of each connected particles. Unfortunately due to the presence of this coupling term, the stability analysis of FSO and the study of its behavior are more complicated. The correspondent continuous form, named Continuous FSO (CFSO), was firstly introduced in [34] and allowed a significant step in the identification of the key conditions for stability analysis. Unfortunately it used a standard ordinary differential equation (ODE) integrator for updating the positions and the velocities of birds and thus it was extremely slow and above all had a high computational cost. The development of closed forms for the fully connected version of CFSO, presented in [35], allowing the updating of the positions and the velocities without requiring numerical integration made the CFSO approach effective and fully useable since it drastically reduced the computational cost. Indeed in the fully connected case all birds are controlled by all and the analytical treatment of the continuous dynamical system is possible. In addition, CFSO showed several features that makes it very interesting and promising such as, among other, the possibility of a supervised approach for tuning parameters [36].

In this paper we present the closed forms for the CFSO in a more general case thanks to the use of circulant matrices: as it will be shown in the appendix this allows us to deal with almost all significant connections between birds. The aim is not only limited to the study of stability but it is extended to the presentation of a continuous model based optimizer as an alternative to the numerical ones. Indeed, this approach has proven to be extremely effective for analyzing stability also in presence of the neighbor-velocity-matching characterizing FSO. Moreover, the proposed optimizer has the peculiarity to be independent from the stochastic updating of parameters, since its exploration and exploitation capabilities can be exalted just by tuning in a deterministic way the value of the parameters on the basis of stability analysis. A comparative analysis among stochastic-numerical and deterministic-continuous algorithms is also presented on classical benchmarks in order to show the advantages of the proposed continuous approach.

2. State equations of continuous flock-of-starlings optimization

In [34,35] it has been proved that it is possible to convert the numerical swarm optimization algorithm, PSO or FSO, into continuous time-domain dynamical kinetic systems described by a set of state-equations. Virtually, the state equations equivalent to the rules used by the swarm-algorithms for updating velocity u_k and position x_k of generic kth particle are:

$$\dot{u}_k(t) = \tilde{\omega}u_k(t) + \tilde{\lambda}(p_{bestk}(t) - x_k(t)) + \tilde{\gamma}(g_{best}(t) - x_k(t)) + \sum_{m=1}^{N}\tilde{h}_{km}u_m(t) \quad (1)$$

$$\dot{x}_k(t) = u_k(t) \quad (2)$$

where the parameters ($\tilde{\omega}$, $\tilde{\lambda}$, $\tilde{\gamma}$, $\tilde{h}_{km}$) are, respectively, the so-called inertial, cognitive and social coefficients. Clearly, $\tilde{h}_{km} \neq 0$ if the kth bird controls the mth one, $\tilde{h}_{km} = 0$ otherwise. In addition, Eq. (1) is useful also for understanding how the global (g_{best}) and personal bests (p_{bestk}) are seen as excitations in the continuous. Indeed, let us assume as excitation of the dynamical system the quantity:

$$\Im_k(t) = \tilde{\lambda} \cdot p_{best\,k}(t) + \tilde{\gamma} \cdot g_{best}(t) \quad (3)$$

and finally, by posing:

$$\tilde{\mu} = \tilde{\lambda} + \tilde{\gamma} \quad (4)$$

the state equation (1) can be written for each k-th particle/bird as follows:

$$\dot{u}_k(t) = \tilde{\omega}u_k(t) - \tilde{\mu}x_k(t) + \sum_{m=1}^{N}\tilde{h}_{km}u_m(t) + \Im_k(t) \quad (5)$$

Eqs. (5) and (2) describe a dynamical system, evolving in the time domain, t, being forced by (3). Then, if we consider a flock made of N birds, the state-equation system has $2N \times 2N$ size and it can be written in a compact form for each jth component of the space solution as follows: