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A new iterative method for solving multiobjective linear programming problem

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ABSTRACT

In the paper we present a new iterative method for solving multiobjective linear programming problems with an arbitrary number of decision makers. The method is based on the principles of game theory. Each step of the method yields a unique solution which respects the aspirations of decision makers within the frame of given possibilities. Each decision maker is assigned an objective indicator which shows the reality of his aspiration and which may be used to define the strategy for the next step. The method can be easily extended to general (nonlinear) multiobjective programming problems but the numerical application would require further research on computational methods.

1. Introduction and motivation

The motivation for this work arises from a practical, frequently encountered problem: Several decision makers (we shall call them *players*) optimize their utilities at the same time and on the same constraint set (*budget*). They can achieve their aspirations at different optimal points. However, only one such point is available and the players are aware of that, which is known as the multiple objective programming problem (MOPP). Solving such a MOPP requires some kind of cooperation among the players. The goal can also be achieved by following the rules of a regulatory subject, if such exists.

The research literature on the cooperative game theory is extensive and our review will include only a select number of papers dealing with the problem of achieving cooperation when choosing the preferred solution in MOPPs. The impact of individual aspirations on promoting the cooperative level has been studied in [9,10,18]. The application of cooperative game theory in social sciences has been presented in [3,11,20,21]. Some specific problems arising from the application of cooperative game theory in physics have been studied in [4,16,17,19].

If the objective functions are linear and the budget is defined by the linear constraints, we are dealing with a multiobjective linear programming problem (MOLPP). There were many attempts at solving MOLPP and a variety of methods have been proposed. All of them require that the players (decision makers) participate in the selection of the preferred solutions. The first wide-ranging review of methods for solving MOLPP appeared in [2], following which many authors developed new methods to remove the shortcomings of the earlier ones. Different methods require different levels of players' participation in the problem solving process. The existing methods are burdened by the complicated solving procedure when a larger number of players are involved.

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Over the last ten years such methods have also incorporated the game theory idea of equilibrium. However, all those methods can only be applied to solving real problems with considerable difficulty. Below is a brief overview of the methods which incorporate cooperative or/and noncooperative game theory ideas.

In [15], a hybrid multiobjective algorithm derived from game theory is applied to an integrated process planning and scheduling (IPPS) problem. Namely, the Nash equilibrium has been used in a game theory based algorithm to deal with multiple objectives. The algorithm is complex and incomprehensible to the players. This is the main disadvantage of the algorithm.

The authors in [1] developed a multiobjective game theory model (MOGM) for balancing economic and environmental concerns in reservoir watershed management and for assistance in decision making. In this case game theory is used as an alternative tool for analyzing the strategic interaction between economic development (land use and development) and environmental protection (water-quality protection and eutrophication control). The methodology is simple enough, but it is only applicable to problems involving two players.

In [5] a production model is considered as a multiobjective linear programming problem with multiple players. It is shown that a multi-commodity game arises from the multi-objective linear production programming problem with multiple decision makers. The characteristic sets in the game were obtained by finding the set of all the Pareto extreme points of the multiobjective programming problem. It is proven that the core of the game is not empty, and points in the core are computed using the duality theory of multiobjective linear programming problems. The least core and the nucleolus of the game are examined as well. The proposed methodology, however, is quite complicated and the players, who should be able to understand it and trust the results, can hardly understand do so.

The authors in [8] develop a multiobjective mixed integer linear programming model, devised to optimize the planning of supply chains using game theory optimization for decision making in cooperative and/or competitive scenarios. The multiobjective problem is solved using the Pareto frontier solutions, and both cooperative and noncooperative scenarios between supply chains were considered. This algorithm is designed for solving a specific problem and it is not generally applicable.

A method for multiobjective categorization based on the game theory and Markov process is proposed in [14]. The authors adopt Shapley value in coalitional games to measure the player's satisfaction degree in a group. Next they present the concept of priority groups and an algorithm to combine small-size priority groups with large-size ones, which may improve the efficiency of calculating the player's satisfaction degree. The complexity of this algorithm is its main disadvantage.

A good application of cooperative and noncooperative game theory has been presented in [12]. A difference between noncooperative and cooperative games is that cooperative game theory admits of binding agreements to choose a joint strategy in the mutual interest of those who agree.

In [13] the authors propose a realistic representation of a decision maker's behavior by synthesizing games-against-nature and goal programming into a single framework. The proposed model has been illustrated by an example from the literature on mathematical programming models for agricultural-decision-making.

The idea of cooperative games is obviously very interesting and can be used for optimization of complex systems with multiple players. In fact, optimization of a complex system requires that the players take into account not only their individual goals but also the need for an optimal functioning of the whole system. The goal of this paper is to use game theory ideas to develop a simple method for solving MOLPP with any number of players.

Generally speaking, the problem to be solved is the following one: optimize of a multi-component system in which each component has its separate goal and the objectives are mutually conflicting. Each component of the system represents one player. In addition to the individual objectives of each system component there is a common goal that cannot be precisely measured: the optimal functioning of the whole system. The system is operating optimally if all components of the system are met at a satisfactory level with regard to the players. The satisfactory level is reached through the process of problem solving, where we use the strengths of game theory, and which allows us to achieve equilibrium solutions. The solving process stops when a desired (Nash) equilibrium is obtained. According to Martin J. Osborne: a Nash equilibrium is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*. (see [7], page 22).

Our aim is to build a simple method for determining optimal solutions MOLPP with multiple players, which uses the good ideas of cooperative games, and where the system operates optimally when all its components operate at a satisfactory level. The players are prepared to cut their aspirations in terms of achieving the goals of system components that they represent, if such behavior will contribute to improving the functioning of the whole system. The solving procedure should lead to the equilibrium solutions (Nash equilibrium).

2. Statement of the problem

In this paper we present a new method which efficiently solves MOLPP. The following problem will be considered:

Let $z_i(x), x \in R^n$, be the given linear objective function for player i (P_i), i.e.

$$z_i(x) = z_i(x_1, x_2, \ldots x_n) = \sum_{j=1}^{n} c_{ij} x_j, \quad i = 1, 2, \ldots, k,$$