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Necessary and sufficient condition for the group consensus of multi-agent systems

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ABSTRACT

This paper focuses on the group consensus issue of multi-agent systems, where the agents in a network can reach more than one consistent values asymptotically. A rotation matrix is introduced to an existing consensus algorithm for single-integrator dynamics. Based on algebraic matrix theories, graph theories and the properties of Kronecker product, some necessary and sufficient criteria for the group consensus are derived, where we show that both the eigenvalue distribution of the Laplacian matrix and the Euler angle of the rotation matrix play an important role in achieving group consensus. Simulated results are presented to demonstrate the theoretical results.

1. Introduction

In recent years, consensus problem of multi-agent systems (MASs) has appeared as a new challenging area of research due to its broad applications in many areas such as computer science, vehicle systems, unmanned air vehicles, scheduling of automated highway systems, formation control of satellite clusters (see, e.g. [1–17] and the references therein). Consensus problem aims to design appropriate protocols and algorithms such that a group of agents can converge to a consistent value.

At present, collective motion from consensus with Cartesian coordinate coupling matrix has received much attention. In 2008, Ren [18] investigated a team of vehicles in the three-dimensional space by introducing a rotation matrix to an existing consensus algorithm for single-integrator kinematics. It was shown that both the network topology and the value of the Euler angle affected the resulting collective motion. In 2009, Ren [19] further extended the results in Ren [18] from single-integrator kinematics to double-integrator dynamics and proved that the network topology, the damping gain and the value of the Euler angle affected the resulting collective motion. However, both Ren [18] and Ren [19] just investigate the normal/traditional consensus issue, i.e., all the agents in a network converge to a common state. As far as we know, few papers are concerned about the group/cluster consensus problem (see, e.g. [7,13–15] and the references therein), where the agents in a network are divided into multiple subgroups and different subgroups can reach different consistent states asymptotically. Compared with the normal consensus, research on group consensus will be more likely to reveal the complexity of multi-agent systems and more accords with the engineering practice. Thus, it is necessary to study the group consensus. Recently, Yu and Wang [14] solved the group average-consensus problem for networks with fixed undirected topology. In 2010, Yu and Wang [7] extended the results in Yu and Wang [14] to networks with both switching topology and communication delay by using double-tree-form transformation, which converted the group consensus problem of continuous-time system into the stability

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of a corresponding reduced system. Requiring that the sum of adjacent weights from every node in one group to all nodes in another group was identical instead of zero, Tian et al. [15] extended the results in Yu and Wang [7]. Based on the Markov chains and nonnegative matrix analysis, Chen et al. [13] established two cluster criteria to deal with the cluster consensus of discrete-time MASs under the assumption that all the weighted factors are nonnegative.

Inspired by the above analysis, this paper will investigate the group consensus of multi-agent model using a distributed protocol with Cartesian coordinate coupling matrix. The main contribution of this paper is to establish some necessary and sufficient group consensus criteria. Specifically, we show that the group consensus can be achieved if and only if the Laplacian matrix $-L$ has two zero eigenvalues, all the nonzero eigenvalues have negative real parts and the Euler angle of the rotation matrix C is below a critical value.

The outline of this paper is as follows. Some basic definitions and supporting results are presented in the next section. Our main results are given in Section 3. Numerical examples are given in Section 4 to illustrate our results. Section 5 concludes this paper.

We use standard notations throughout this paper. M^T, M^{-1}, $rank(M)$ represent the transpose, inverse and rank of the matrix M, respectively. $arg(.)$ denotes the phase of a number. $\otimes$ denotes the Kronecker product. $1_n = [1, \dots, 1]^T \in \mathbb{R}^n$ and 0 represents any zero matrix with appropriate dimension. I_n denotes the $n \times n$ identity matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulations and preliminaries

In this section, some basic knowledge on graph theory, problem formulations, some definitions and lemmas are given as the preliminaries of this paper.

2.1. Graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted directed graph of order $n+m$ with the set of nodes $\mathcal{V}=\{v_1, v_2, \dots, v_{n+m}\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the nonsymmetric weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(n+m)\times(n+m)}$ with real adjacency elements a_{ij}. The node indexes belong to a finite index set $\ell = \{1, 2, \dots, n+m\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_j, v_i)$. The adjacency elements associated with the edges of the graph are nonzero, i.e., $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} \neq 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \ell$. The set of neighbors of node v_i is denoted by $N_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. $L(\mathcal{G}) = [l_{ij}]$ is the Laplacian of topology $\mathcal{G}$, and is defined by

$$l_{ij} = \begin{cases} -a_{ij}, & j \neq i, \\ \sum_{k=1,k\neq i}^{n+m} a_{ik}, & j = i. \end{cases}$$

A directed path from node v_i to v_j is a sequence of edges $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \dots, (v_{i_l}, v_j)$ in the directed graph with distinct nodes v_{i_k}, $k = 1, 2, \dots, l$. A root r is a node having the property that for each node v different from r, there is a directed path from r to v. A directed tree is a directed graph, in which there is exactly one root and every node except for this root has exactly one parent node. A directed spanning tree is a directed tree, which consists of all the nodes and some edges in $\mathcal{G}$.

2.2. Existing consensus algorithm for single-integrator dynamics

Suppose that each agent has the dynamics given by

$$\dot{r}_i(t) = u_i(t), \quad \forall i \in \ell, \tag{1}$$

where $r_i(t) := \begin{bmatrix} x_i(t) \\ y_i(t) \\ z_i(t) \end{bmatrix} \in \mathbb{R}^3, u_i(t) \in \mathbb{R}^3$ are, respectively, the state and control input associated with the ith agents.

For system (1), design the following protocol

$$u_i(t) = \sum_{v_j \in N_i} a_{ij} C(r_j(t) - r_i(t)), \tag{2}$$

where $a_{ij} \geqslant 0, \ \forall i, j \in \ell$, and the matrix $C \in \mathbb{R}^{3\times 3}$ denotes a rotation matrix. Define $r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{n+m} \end{bmatrix}$ and substitute (2) into (1), then system (1) can be written in a vector form as

$$\dot{r}(t) = -(L \otimes C) r(t), \tag{3}$$