Applied Mathematics and Computation 243 (2014) 923–935

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Improved stability criteria for a class of neural networks with variable delays and impulsive perturbations

Chang-Bo Yang [a,b], Ting-Zhu Huang [a,*]

[a] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China
[b] Institute of Nonlinear Analysis, Kunming University, Kunming, Yunnan 650214, PR China

ARTICLE INFO

Keywords:
Neural networks
Exponential stability
M-matrix
Variable delays
Impulses

ABSTRACT

This paper further investigates the problem of stability analysis for a kind of neural networks with variable delays and nonlinear impulses. With the help of *M*-matrix, the homeomorphism theory, some effective inequalities and analysis techniques, certain novel criteria on the existence, uniqueness and exponential stability of the equilibrium point have been established. Moreover, these criteria possess adjustable real parameters, which extend and improve many existing results in the literature. In the end, two numerical examples are provided to illustrate the validation of the theoretical results.

1. Introduction

Over the past few decades, several kinds of neural networks [1–3] such as Hopfield neural networks, cellular neural networks, bidirectional associative memory neural networks (for short by HNNs, CNNs, BAMNNs, respectively) have received considerable attentions because of their great potential applications in areas of signal processing, pattern recognition, associative memory and optimization [4,5]. As we know, during the hardware implementation of neural networks, time delays are inevitable due to finite switching speeds of the amplifiers and communication time, which may bring about complex influence on the system such as oscillation and instability [6]. On the other hand, impulsive effects widely exist in many realistic networks [7,8], which may be caused by witching phenomenon, sudden changes, or other unexpected noise. Therefore, it is more appropriate to take both delays and impulsive effects into account when modeling neural networks, and lots of important stability results have been reported on HNNs, CNNs, BAMNNs with delays and impulses. One can refer to [9–21] and references therein.

Recently, authors in [22] introduced a neural network of general type, which incorporated impulsive delayed HNNs, CNNs, BAMNNs as its special cases by choosing suitable nonlinear terms $f_i(\bullet,\bullet)$ and was formulated by the following form:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + f_i(x_1(t),\ldots,x_n(t),x_1(t-\tau_{i1}),\ldots,x_n(t-\tau_{in})) + I_i, \\ \qquad t>0, \quad t\neq t_k, \quad i\in\mathbb{N}\triangleq\{1,2,\ldots,n\}, \\ \triangle x_i(t_k) = x_i(t_k) - x_i(t_k^-) = J_{ik}(x_i(t_k^-)), \quad k\in\mathbb{Z}\triangleq\{1,2,\ldots\}. \end{cases} \tag{1.1}$$

where the impulsive moments are such that $0=t_0<t_1<t_2<\ldots$ and $\lim_{t\to\infty} t_k=+\infty$. $x_i(t)$ are the state of neurons and $c_i>0$ denote the passive decay rates. $I=(I_1,I_2,\ldots,I_n)^T$ are the constant input vector. Under the constant delays case and the linear impulsive disturbances to equilibrium point, some results on stability of model (1.1) have been obtained by

* Corresponding author.
E-mail addresses: cbyang348@126.com (C.-B. Yang), tingzhuhuang@126.com (T.-Z. Huang).

http://dx.doi.org/10.1016/j.amc.2014.06.045
0096-3003/

Lyapunov functional method and spectral theory of matrix. However, it is an idealized assumption that the delays are invariable. Actually, the delays are time-varying [23–27], and the nonlinear impulsive perturbations are more commonly encountered in the real environment [28]. Due to these factors, we shall modify model (1.1) as follows:

$$\begin{cases} \dot{x}_i(t) = -b_i(x_i(t)) + f_i(x_1(t),\dots,x_n(t),x_1(t-\tau_{i1}(t)),\dots,x_n(t-\tau_{in}(t))) + I_i, \\ \qquad\qquad t > 0, \quad t \neq t_k, \quad i \in \mathbb{N} \triangleq \{1,2,\dots,n\}, \\ x_i(t_k) = h_{ik}(x_1(t_k^-),\dots,x_n(t_k^-)) + \gamma_{ik}, \quad k \in \mathbb{Z} \triangleq \{1,2,\dots\}, \\ x_i(s) = \phi_i(s), \quad s \in [-\tau, 0], \end{cases} \tag{1.2}$$

where $b_i(\cdot),\ f_i(\bullet,\bullet),\ h_{ik}(\bullet)$ are the general nonlinear functions, which satisfy certain conditions defined later (see (A_1)–(A_3), $\tau_{ij}(t)$ are the transition delays, which are variable and bounded with $0 \leqslant \tau_{ij}(t) \leqslant \tau_{ij}\ (i,j \in \mathbb{N})$ and γ_{ik} are the external impulsive inputs at time t_k.

It is well known that stability analysis of neural networks is a prerequisite for their applications. For instance, in solving optimization problems [29,30], the neural network must be designed to have one unique and globally stable equilibrium point and there is a strong motivation to study the stability of neural networks. Until now, numerous stability criteria have been established for variants of neural networks via different approaches [31–42]. In particular, in [31–36], several good results are presented by the construction of Lyapunov functionals and the linear matrix inequality (LMI) techniques. It is observed that, if the neural network model can be written in the vector–matrix form, then the LMI is an effective tool because the obtained conditions are in terms of LMI and its validity can be easily checked by the LMI toolbox in Matlab. Since the transition delays $\tau_{ij}(t)$ are different and the impulsive functions $h_{ik}(\bullet)$ may be nonlinear in reality, the system (1.2) is not be conveniently expressed by the vector–matrix form. So the LMI is not an appropriate approach to the stability analysis of system (1.2). To best our knowledge, the M-matrix is also an important tool in stability analysis of neural networks. Inspired by [37–42], we shall carry out a further analysis on stability of system (1.2) by M-matrix theory. The novelty of this paper lies in combining a general norm $\|\cdot\|_{\{d,r,\infty\}}$, some nice-established inequalities and analysis techniques, a general M-matrix-based criterion on the existence, uniqueness and stability of equilibrium point of system (1.2) with impulses has been derived, which unifies and improves many previous works in [16,17,22–24,27,37,43–45]. Moreover, the impulsive parts in system (1.2) are characterized by the general nonlinear functions, and this endows our results with wider applicability in real life problems.

The rest of this paper is organized as follows. In Section 2, some notations, conditions, definitions and important lemmas are presented. In Section 3, some improved criteria are established for the existence, uniqueness and global exponential stability of equilibrium point of system (1.2). In Section 4, two comparative examples with numerical simulations are given. Finally, some conclusions are summarized in Section 5.

2. Preliminaries

In this section, we begin with some standard notations. Let $\mathfrak{R}$ and $\mathfrak{R}^n$ be the set of real numbers and n-dimensional vector space, respectively. The symbol $(\cdot)^T$ stands for the transpose of a vector or matrix. For any $x = (x_1, x_2, \dots, x_n)^T \in \mathfrak{R}^n$, its norm is defined by $\|x\|_{\{d,r,\infty\}} = \max_{1 \leqslant i \leqslant n}\left\{d_i^{-\frac{1}{r}}|x_i|\right\}$, where $r \geqslant 1,\ d_i > 0,\ i \in \mathbb{N}$. Clearly, the usual maximum norm $\|\cdot\|_\infty$ in $\mathfrak{R}^n$ is a special case of $\|\cdot\|_{\{d,r,\infty\}}$. We also use $\mathcal{C} \triangleq C([-\tau,0],\mathfrak{R}^n)$ to denote a set of all continuous functions from $[-\tau,0]$ to $\mathbb{R}^n$, where $\tau = \max_{1\leqslant i,j\leqslant n}\{\tau_{ij}\}$. For any $\phi(s) \in \mathcal{C}$, equipped with its induced norm $\|\phi\| = \sup_{s\in[-\tau,0]}\|\phi(s)\|_{\{d,r,\infty\}}$, then $\mathcal{C}$ is a Banach space.

Furthermore, we make the following conditions:

(A_1) (Sign condition.) For $i \in \mathbb{N}$, each behaved function $b_i(\cdot) : \mathfrak{R} \to \mathfrak{R}$ is continuous and there exists a constant $\gamma_i > 0$ such that

$$sign(x-y)(b_i(x) - b_i(y)) \geqslant \gamma_i|x-y|,$$

where $x,\ y \in \mathfrak{R}$.

(A_2) (Lipschitz-type condition.) For $i \in \mathbb{N}$, each activation function $f_i(\bullet,\bullet) : \mathfrak{R}^n \times \mathfrak{R}^n \to \mathfrak{R}$ is continuous and there exist constants $\alpha_{ij} > 0$ and $\beta_{ij} > 0$ such that

$$|f_i(u_1,\dots,u_n,v_1,\dots,v_n) - f_i(\tilde{u}_1,\dots,\tilde{u}_n,\tilde{v}_1,\dots,\tilde{v}_n)| \leqslant \sum_{j=1}^n \alpha_{ij}|u_j - \tilde{u}_j| + \sum_{j=1}^n \beta_{ij}|v_j - \tilde{v}_j|,$$

where $u = (u_1,\dots,u_n)^T,\ v = (v_1,\dots,v_n)^T,\ \tilde{u} = (\tilde{u}_1,\dots,\tilde{u}_n)^T,\ \tilde{v} = (\tilde{v}_1,\dots,\tilde{v}_n)^T \in \mathfrak{R}^n$.

(A_3) (Lipschitz-type condition.) For $i \in \mathbb{N}, k \in \mathbb{Z}$, each impulsive function $h_{ik}(\bullet) : \mathfrak{R}^n \to \mathfrak{R}$ is continuous and there exists a constant $\zeta_{ij}^{(k)} > 0$ such that

$$|h_{ik}(u_1,\dots,u_n) - h_{ik}(\tilde{u}_1,\dots,\tilde{u}_n)| \leqslant \sum_{j=1}^n \zeta_{ij}^{(k)}|u_j - \tilde{u}_j|,$$

where $u = (u_1,\dots,u_n)^T,\ \tilde{u} = (\tilde{u}_1,\dots,\tilde{u}_n)^T \in \mathfrak{R}^n$.