



Global stability of a stochastic predator–prey system with infinite delays [☆]



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ABSTRACT

This paper is concerned with the global asymptotic stability of a stochastic delay predator–prey system with Beddington–DeAngelis functional response. Sufficient criteria for the global asymptotic stability of the system are established. Some simulation figures are provided to show that our model is more realistic than existing models.

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1. Introduction

In a natural ecosystem, fundamental features of population interactions, such as predation and competition have been elucidated by empirical and theoretical investigations of the dynamics between two species. One significant component of the predator–prey relationship is the predator's functional response. Skalski and Gilliam [23] stated that the functional response can provide better descriptions of predator feeding over a range of predator–prey abundances by comparing the statistical evidence from 19 predator–prey systems with the three predator-dependent functional responses (Hassell–Varley, Beddington–DeAngelis and Crowley–Martin), and in some cases, the Beddington–DeAngelis type functional response performed even better.

The original predator–prey system with Beddington–DeAngelis type functional response is

$$\frac{dx}{dt} = x \left[b_1 - a_{11}x - \frac{a_{12}y}{1 + \beta x + \gamma y} \right], \quad \frac{dy}{dt} = y \left[-b_2 - a_{21}y + \frac{a_{22}x}{1 + \beta x + \gamma y} \right], \quad (1)$$

where $x = x(t)$ and $y = y(t)$ stand for the prey's and the predator's densities at time t , respectively. For biological significance of each coefficient we refer the reader to [2,4].

On the other hand, population dynamics is always affected by environmental noise (see e.g. [5,6]), which is an important component in an ecosystem. Thus many authors studied stochastic models which corresponding to deterministic models to reveal the influence of environmental noise on the population dynamics (see e.g. [1,3,9,18,19,15–17,21,22,24,10,12]). Moreover, recently, Liu and Han [14,13] have studied integro-differential equations with impulsive integral conditions, their work extend some existing models to a great extent of this field.

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In 2011, Liu and Wang [11] studied the following nonlinear stochastic predator–prey system

$$\begin{cases} dx = x \left[1 - a_{11}x - \frac{a_{12}y}{1+\beta x+\gamma y} \right] [b_1 dt + \sigma_1 dB_1(t)], \\ dy = y \left[-1 + \frac{a_{21}x}{1+\beta x+\gamma y} - a_{22}y \right] [b_2 dt + \sigma_2 dB_2(t)], \end{cases} \tag{2}$$

with the initial value $x(0) = x_0 > 0$, $y(0) = y_0 > 0$. They showed that if the positive equilibrium point of the deterministic system was globally stable, then the stochastic model would preserve the nice property provided the noise was small enough. However, like it or not, time delays frequently occur in almost every situation, Kuang [8] has revealed that ignoring time delays means ignoring reality. Hence it is essential to take time delays into account. Motivated by these, in this paper, we will propose and study a more realistic and complex stochastic delay predator–prey system with Beddington–DeAngelis functional response

$$\begin{cases} dx(t) = x(t) \left[r_1 - a_{11}x(t) - a_{12}x(t - \tau_1) - a_{13} \int_{-\infty}^0 x(t + \theta) d\mu_1(\theta) - \frac{a_{14}y(t)}{1+\beta x(t)+\gamma y(t)} \right] dt + \sigma_1 x(t)(x(t) - x^*) dB_1(t), \\ dy(t) = y(t) \left[-r_2 - b_{11}y(t) - b_{12}y(t - \tau_2) - b_{13} \int_{-\infty}^0 y(t + \theta) d\mu_2(\theta) + \frac{b_{14}x(t)}{1+\beta x(t)+\gamma y(t)} \right] dt + \sigma_2 y(t)(y(t) - y^*) dB_2(t), \end{cases} \tag{3}$$

where σ_i^2 represents the intensity of the noise, $B_i(t)$ is a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t \in \mathbb{R}_+}$ satisfying the usual conditions, $\tau_i \geq 0$ and $\mu_i(\theta)$ is a probability measure on $(-\infty, 0]$, $i = 1, 2$, (x^*, y^*) is the equilibrium state of system (3). For more biological motivation on this type of modeling we refer the reader to Gard [5,6].

This paper is organized as follows: in Section 2, we will show that if the noise is small enough and the positive equilibrium state of the deterministic model is globally stable, the stochastic system will keep the property. In Section 3, we will introduce some numerical simulations to support our main result. Finally we give some conclusions and discussions.

2. Global stability

As the biological significance of $x(t)$ and $y(t)$ in model (3), we should firstly give some conditions under which system (3) has a unique global positive solution.

Lemma 2.1. Consider model (3), if $\sigma_i > 0$, $i = 1, 2$, then there is a unique positive local solution $(x(t), y(t))$ on $(-\infty, \tau_e)$ almost surely (a.s.) for any given initial value $(\zeta_1, \zeta_2) \in BC((-\infty, 0]; \mathbb{R}_+^2)$, where τ_e denotes the explosion time and $BC((-\infty, 0]; \mathbb{R}_+^2)$ represents the family of bounded and continuous functions from $(-\infty, 0]$ to \mathbb{R}_+^2 with the norm $\|\zeta_i\| = \sup_{\theta \leq 0} |\zeta_i(\theta)|$, $i = 1, 2$.

Because the proof of this lemma is similar to that in [11], we omit it here.

Lemma 2.2. Consider system (3), there is a unique global solution $(x(t), y(t))$ on \mathbb{R} for any given initial value $(\zeta_1, \zeta_2) \in BC((-\infty, 0]; \mathbb{R}_+^2)$ and the solution will remain in \mathbb{R}_+^2 with probability 1, where $\mathbb{R}_+^2 = \{x \in \mathbb{R}^2 | x_i > 0, i = 1, 2\}$.

Proof. The following proof is motivated by the work of Liu and Wang [11]. Let

$$M(x, y) = \frac{a_{14}y(t)}{1 + \beta x(t) + \gamma y(t)}, \quad N(x, y) = \frac{b_{14}x(t)}{1 + \beta x(t) + \gamma y(t)}.$$

Obviously, if $(x, y) \in \mathbb{R}_+^2$, then $0 \leq M(x, y) \leq \frac{a_{14}}{\gamma}$, $0 \leq N(x, y) \leq \frac{b_{14}}{\beta}$.

Let $k_0 > 0$ be large enough such that $\zeta_i(\theta)$ ($i = 1, 2$), $\theta \in (-\infty, 0]$ lying within the interval $[1/k_0, k_0]$. For every integer $k > k_0$, define the stopping time

$$\tau_k = \inf \{t \in (-\infty, \tau_e) : x(t) \notin (1/k, k) \text{ or } y(t) \notin (1/k, k)\},$$

Obviously, τ_k is increasing as $k \rightarrow \infty$. Let $\tau_\infty = \lim_{k \rightarrow \infty} \tau_k$, then we have $\tau_\infty \leq \tau_e$ a.s. Next, we only need to verify $\tau_\infty = \infty$. If this claim is not true, then there are two constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $P\{\tau_\infty \leq T\} > \varepsilon$. So there is an integer $k_1 \geq k_0$ such that

$$P\{\tau_k \leq T\} \geq \varepsilon, \quad k \geq k_1. \tag{4}$$

Define

$$V(x, y) = (\sqrt{x} - 1 - 0.5 \ln x) + (\sqrt{y} - 1 - 0.5 \ln y).$$

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