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Permanence of a stage-structured predator–prey system with impulsive stocking prey and harvesting predator ☆

Xinhui Wang, Canyun Huang *

Institute of Applied Mathematics, Lanzhou University of Technology, Lanzhou, Gansu 730050, People's Republic of China

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ABSTRACT

In this paper we propose a stage-structured predator prey system with impulsive stocking immature prey and harvesting predator at different moment. We analyze the global attractivity of the mature prey-extinction periodic solution, and obtain sufficient conditions for the permanence of the system. Numerical simulations are also inserted to verify the feasibility of the theoretical results. Moreover, the obtained results show that impulsive stocking immature prey or harvesting predator may play a key role on the permanence of the system and provide tactical basis for the biological resources management.

1. Introduction

Since biological resources can reproduce themselves in a natural way, it is regarded as the most important part of the natural resources. In recent years, how to get maximal profit and protect the sustainable development of biological system have already drawn a great deal of attention of biologists and resources economists [1]. Recent literatures [2–4] revealed that harvesting and stocking prey or predator can influence ecological system, but overusing these methods sometimes can destroy the ecological balance of renewable resources. To protect the permanence of biological resources, it will be a dominant theme to seek for the suitable threshold of harvesting and stocking in mathematical ecology. Compared with the growth process of biological resources, manual harvesting or stocking always happen in a short time, which should be described by impulsive perturbation instead of continuous effect. Consequently, impulsive differential equations provide a more realistic description of such exploited system [12,9–11,5,8,6,7]. Recently, impulsive differential equations have been extensively used as models in biology, physics, chemistry, engineering and other sciences, with particular emphasis on population dynamics [13–17]. In [12], Wang proposed a system with impulsive immigration of predator. The effect of harvesting on prey and predator with constant efforts has been discussed by Negi [11]. In [18], Shao and Li discussed an impulsive predator–prey system with stage structure and generalized functional response, to model the diffusion by impulses. Sufficient conditions are established for the existence of a predator-extinction positive periodic solution and the permanence of the system. Numerical simulation shows that impulses and functional response affect the dynamics of the system.

In the natural world, many species usually go through two distinct life stages from birth to death, immature and mature. The immature takes τ units of time to mature and it is well-known that time delay τ is an important factor of mathematical models in ecology. The predator–prey models with stage-structure have been investigated by many researchers [19–24]. Time delay and impulse are incorporated into predator–prey models, which greatly enriches biologic background. But the system become so complicated that it causes us greatly difficulties to study the model. The investigation of impulsive delay

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* Corresponding author.
E-mail address: canyun_h@sina.com (C. Huang).

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differential equations is inchoate, and focus on theoretical analysis [26,25,27]. In [9] Liu studied a stage-structured model with impulsive perturbations on predator. The recent works of Jiang [8] and Jiao [6,7] considered some delayed predator–prey models with impulsive stocking on prey and continuous harvesting on predator, in which the influences of impulses and stage-structure are analyzed.

The dynamical relationships between predator and prey can be represented by the functional response, which refers to the change in the density of prey attached per unit time per predator as the prey density change. When the prey are simple algology cell of animal, invertebrate and vertebrate, Holling type-I, Holling type-II and Holling type-III functional response are proposed, respectively. Luo [10] considered a stage-structured predator–prey Leslie–Gower Holling II model with disturbing pulse. In [5], the authors analyzed a stage-structured predator–prey model with Holling mass defence, and discussed the effect of impulsive stocking prey and harvesting predator at same fixed time. In 1975, Beddington [29] and DeAngelis et al. [30] introduced an additional functional response

$$\varphi(x) = \frac{mx}{a + x + by}.$$

This functional response has an extra term in the denominator, which models the mutual interference between predators and avoids the "low densities problem" of the ratio-dependent type functional response. In [31], an impulsive delayed predator–prey model with stage-structure and Beddington-type functional response is established, in which harvesting on prey and stocking on predator are mostly stressed. In fact, stocking prey and harvesting predator usually be at different moment in the real world. The paper [32] established a Lotka–Volterra model with birth pulse and impulsive catching or poisoning for the prey at different moment, and obtained sufficient conditions of the global attractivity of predator-extinction periodic solution and the permanence of the system. Motivated by these literatures [6–8,22,28,31–34], we consider the following impulsive stage-structured differential system

$$\begin{cases} \left.\begin{array}{l} x_1'(t) = r_1x_2(t) - r_1e^{-w\tau}x_2(t-\tau) - \gamma x_1(t), \\ x_2'(t) = r_1e^{-w\tau}x_2(t-\tau) - \frac{cx_2(t)y(t)}{\alpha+x_2(t)+\beta y(t)} - d_1x_2(t) - d_2x_2^2(t), \\ y'(t) = r_2y(t) + \frac{kcx_2(t)}{\alpha+x_2(t)+\beta y(t)}y(t) - d_3y^2(t), \end{array}\right\} t \neq (n+l-1)T, nT, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t) + \mu, \\ x_2(t^+) = x_2(t), \\ y(t^+) = y(t), \end{array}\right\} t = (n+l-1)T, \\ \left.\begin{array}{l} x_1(t^+) = x_1(t), \\ x_2(t^+) = x_2(t), \\ y(t^+) = (1-p)y(t), \end{array}\right\} t = nT, \end{cases} \tag{1.1}$$

where $x_1(t), x_2(t)$ represent the immature and mature prey densities respectively, $y(t)$ denotes the density of the predator, τ represents a constant time to maturity, and $r_1, w, c, \alpha, \beta, d_1, d_2, r_2, k, d_3$ are positive constants. r_1 is the birth rate of the immature prey, $w(w > d_1), d_1$ denotes the mortality rate of the mature prey, c is the maximum numbers of the mature prey that can be eaten by a predator per unit of time, d_2, d_3 are the intra-specific competition rate of the mature prey and the predator, r_2 is the intrinsic growth rate of the predator, k is the rate of conversing prey into predator, $\mu \geqslant 0$ is the stocking amount of the immature prey at $t = (n+l-1)T$, l is a constant $(0 < l < 1)$, $p(0 \leqslant p < 1)$ represents the harvesting rate on the predator at $t = nT, n \in Z_+$ and $Z_+ = \{1, 2, \ldots\}$, T is the seasonal period of the biological resources.

The initial conditions for system (1.1) are

$$(\phi_1, \phi_2, \phi_3) \in C\left([-\tau, 0], R_+^3\right), \quad \phi_i(0) > 0, i = 1, 2, 3, \quad R_+^3 = \left\{x \in R^3 : x \geqslant 0\right\}.$$

From the biological point, we only consider system (1.1) in the following region

$$D = \{(x_1, x_2, y) : x_1 \geqslant 0, \ x_2 \geqslant 0, \ y \geqslant 0\}.$$

The reminder of this paper is arranged as follows. In Section 2, some fundamental notations, definitions and lemmas are firstly given. In Section 3, we consider the global attractivity of the mature prey-extinction periodic solution. In Section 4, we investigate the sufficient conditions for the permanence of system (1.1). Numerical simulations are presented to verify the feasibility of theoretical results in last section.

2. Notations and preliminaries

Let $R_+ = [0, +\infty), R_+^3 = \{x \in R^3, x \geqslant 0\}$. The solution of (1.1), denoted by $x(t) = (x_1(t), x_2(t), y(t))^T$, is a piecewise continuous function $x : R_+ \to R_+^3, x(t)$ is continuous on $((n-1)T, (n+l-1)T]$ and $((n+l-1)T, nT], x((n-l+1)T^+) = \lim_{t \to (n-l+1)T^+} x(t)$ and $x(nT^+) = \lim_{t \to nT^+} x(t)$ exist. Obviously, the global existence and uniqueness of solution of (1.1) is guaranteed by the smoothness properties of f, which denotes the right-side of the first three equations of system (1.1) (see [35,36]).