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Improved estimation of population variance using information on auxiliary attribute in simple random sampling

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ABSTRACT

Singh and Kumar (2011) [19] suggested estimators for calculating population variance using auxiliary attributes. This paper proposes a family of estimators based on an adaptation of the estimators presented by Kadilar and Cingi (2004) [5] and Singh et al. (2007) [16], and introduces a new family of estimators using auxiliary attributes. The expressions of the mean square errors (MSEs) of the adapted and proposed families are derived. It is shown that adapted estimators and suggested estimators are more efficient than Singh and Kumar (2011) [19] estimators. The theoretical findings are supported by a numerical example.

1. Introduction

It is well known that the auxiliary information in the theory of sampling is used to increase the efficiency of estimator of population parameters. Out of many ratio, regression and product methods of estimation are good examples in this context. There exist situations when information is available in the form of attribute which is highly correlated with y. Taking into consideration the point bi-serial correlation coefficient between auxiliary attribute and study variable, several authors including Naik and Gupta [11], Jhajj et al. [4], Shabbir and Gupta [13], Singh et al. [16–18], Abd-Elfattah et al. [1], Malik and Singh [8], Malik and Singh [9], Malik and Singh [10], Singh [15], Singh and Malik [20] and Sharma et al. [14] defined ratio estimators of population mean when the prior information of population proportion of units, possessing the same attribute is available.

In many situations, the problem of estimating the population variance σ^2 of study variable y assumes importance. When the prior information on parameters of auxiliary variable(s) is available, Das and Tripathi [2], Isaki [3], Prasad and Singh [12], Kadilar and Cingi [6], Kadilar and Cingi [7] and Singh et al. [16] have suggested various estimators of S_y^2.

Consider a sample of size n drawn by SRSWOR from a population of size N. Let y_i and φ_i denote the observations on variable y and φ respectively for the ith unit ($i = 1,2,3,\ldots,N$). It is assumed that attribute φ takes only the two values 0 and 1 according as $\varphi = 1$, if ith unit of the population possesses attribute $\varphi = 0$, if otherwise.

The variance of the usual unbiased estimator $\hat{S}_y^2$ is given by

$$\mathrm{var}(\hat{S}_y^2) = \frac{S_y^4}{n}(\lambda_{40} - 1) \tag{1.1}$$

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where, $\lambda_{rq} = \frac{\mu_{rq}}{(\mu_{20}^{r/2}\mu_{02}^{q/2})}$, $\mu_{rq} = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^r(\varphi_i - P)^q}{(N-1)}$.

In this paper we have proposed a family of estimators for the population variance S_y^2 when auxiliary variable is in the form of attribute. For main results we confine ourselves to sampling scheme SRSWOR ignoring the finite population correction.

2. Estimators in literature

In order to have an estimate of the study variable y, assuming the knowledge of the population proportion P, Singh and Kumar [19] proposed the following estimators

$$t_1 = s_y^2 \frac{S_\phi^2}{s_\phi^2} \tag{2.1}$$

$$t_2 = s_y^2 + b_\varphi\left(S_\varphi^2 - s_\varphi^2\right) \tag{2.2}$$

$$t_3 = s_y^2 \exp\left[\frac{S_\varphi^2 - s_\varphi^2}{S_\varphi^2 - s_\varphi^2}\right] \tag{2.3}$$

The MSE expression of the estimator t_1 and variance of t_2 are given, respectively, by

$$\mathrm{MSE}(t_1) = \frac{S_y^4[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)]}{n} \tag{2.4}$$

$$V(t_2) = \frac{1}{n}\left[S_y^4(\lambda_{40}-1) + b_\phi^2 S_\varphi^2(\lambda_{04}-1) - 2b_\phi S_y^2 S_\varphi^2(\lambda_{22}-1)\right] \tag{2.5}$$

On differentiating (2.5) with respect to b and equating to zero we obtain

$$b_\varphi = \frac{S_y^2(\lambda_{22}-1)}{S_x^2(\lambda_{04}-1)} \tag{2.6}$$

Substituting the optimum value of b_φ in (2.5), we get the minimum variance of the estimator t_2, as

$$\min .V(t_2) = \frac{S_y^4}{n}\left[(\lambda_{40}-1) - \frac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)-1}\right] \tag{2.7}$$

The MSE expression of the estimator t_3 is given by

$$\mathrm{MSE}(t_3) = \frac{S_y^4}{n}\left[(\lambda_{40}-1) + \frac{(\lambda_{04}-1)}{4} - (\lambda_{22}-1)\right] \tag{2.8}$$

3. The adapted estimators

Following Kadilar and Cingi [5], we propose the following variance estimators using known values of some population parameter(s),

$$t_{KC1} = s_y^2\left(\frac{S_\varphi^2 + C_p}{s_\varphi^2 + C_p}\right) \tag{3.1}$$

$$t_{KC2} = s_y^2\left(\frac{S_\varphi^2 + \beta_{2\varphi}}{s_\varphi^2 + \beta_{2\varphi}}\right) \tag{3.2}$$

$$t_{KC3} = s_y^2\left(\frac{S_\varphi^2\beta_{2\varphi} + C_p}{s_\varphi^2\beta_{2\varphi} + C_p}\right) \tag{3.3}$$

$$t_{KC4} = s_y^2\left(\frac{S_\varphi^2 C_p + \beta_{2\varphi}}{s_\varphi^2 C_p + \beta_{2\varphi}}\right) \tag{3.4}$$

where, s_y^2 and s_ϕ^2 are unbiased estimator of population variances S_y^2 and S_ϕ^2, respectively.