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Estimation of near surface wind speeds in strongly rotating flows

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ABSTRACT

Modeling studies consistently demonstrate that the most violent winds in tornadic vortices occur in the lowest tens of meters above the surface. These velocities are unobservable by radar platforms due to line of sight considerations. In this work, a methodology is developed which utilizes parametric tangential velocity models derived from Doppler radar measurements, together with a tangential momentum and mass continuity constraint, to estimate the radial and vertical velocities in a steady axisymmetric frame. The main result is that information from observations aloft can be extrapolated into the surface layer of the vortex. The impact of the amount of information available to the retrieval is demonstrated through some numerical tests with pseudo-data.

1. Introduction

The strongest wind speeds in tornados are believed to occur a few tens of meters above the surface. Due to line of sight limitations, radar platforms are typically unable to measure this portion of the atmosphere. The relationship between the measurable flow aloft, and the unobservable (by radar) flow near the surface is complex (see for instance [2,8,10,12] for different flow regimes).

The reviews [11,13,15] discuss the dynamics of different sections of a tornado. Snow [15] describes the change in magnitude of the different wind components both in the vertical and radial directions, which is based on simulations in fluid dynamics models and in the Tornado Vortex Chamber [1] at Purdue University. A tornado with a positive vertical velocity along the central axis is called a "single celled" vortex. The tangential velocity mean field increases as a function of height from ground level to a maximum, and then decreases again to the top of the vortex. Similarly, the tangential velocity increases as a function of the distance from the center of the vortex until it reaches a maximum, and then decreases to zero. This behavior can be captured with empirical parametric models, such as those discussed in [17]. Models of this type have also been used in observational studies such as [18] to better understand measurements in the presence of noisy observations.

In this paper, we estimate the three components of the wind velocity near ground level from observations aloft. The paper is divided into sections as follows. In Section 2, we review the basic considerations regarding observations of atmospheric circulations by radar instruments and define the problem domain and relevant parameters of interest. Section 3 introduces a method for estimating the vortex radial and vertical velocities, and Section 4 discusses the mathematical issues related to

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this method. The mathematical issues include positive aspects of estimating flow fields with these dynamics, as well as situations in which the dynamics are insufficient to estimate the flow on the entire domain. Section 5 examines a few physical limitations of the approach. In Section 6, we perform an identical twin experimental test of the method for a tornado-like vortex. We generate pseudo-observations with an assumed tangential velocity model and random errors. Then we estimate the flow using the same tangential velocity model. This test is not meant to prove conclusively that the method will work with a real data set, but rather to show the theory in action.

Remark 1.1. Many researchers in meteorology currently use variational techniques to estimate wind fields from radar velocity measurements. These techniques are powerful, and are especially useful for dealing with noisy measurements. They face the problems common to all optimal estimation techniques. Some of the difficulties are finding a unique global minimum and minimizer, and the tendency of least squares techniques to reduce the magnitude of smaller scale features. Further, a minimizer of a set of weakly enforced constraints may not satisfy any of the constraints particularly well. Boundary conditions for these types of methods are usually not chosen physically, but rather are allowed to be retrieved with the rest of the variables. The authors are well acquainted with these techniques, and propose the techniques in this paper as a first step toward remedying some of these difficulties. Most variational techniques utilize some sort of descent based minimization procedure, and the solutions provided by the method in this paper could be used as the "first guess" which is required of all iterative schemes.

2. Background

Assume that two radar instruments measure a given volume of air simultaneously. The two horizontal components of the velocity can be recovered if the radar beams are approximately horizontal. In this case, the measurements contain very little information about the vertical component of velocity, i.e. are orthogonal to the vertically pointing basis vector. Take the flow to be in cylindrical coordinates, with the axis of the coordinate system aligned along the vertical axis of the vortex. Thus the recovered components are the tangential and radial components of the swirling flow.

For the remainder of the this work, assume two sets of wind measurements, which have been converted to radial and tangential velocities for the vortex of interest, and averaged azimuthally to create an axisymmetric mean pair of velocities. The spatial domain includes the vertical axis and the surface and measurements which are representable by a parametric model. A family of parametric models for the tangential velocity is chosen which best approximate the qualitative features of the given data, then a particular parameter set is selected so that the tangential velocity model is optimal (in some sense). This is done *in advance* of seeking to estimate u and w.

In the next section, the estimation of radial and vertical velocities in a layer near the surface, where the velocities are not observable, is considered. The problem is posed on the domain Ω, which is illustrated in Fig. 1. The domain is decomposed into an observable region Ω_o and an unobservable region Ω_h, separated by a horizontal line $z = h$. This line is referred to as the *minimum observable height* (MOH) line. The domain on which we interested in retrieving the flow is referred to as the *surface layer*, which is the portion of the domain between the height $z = 0$ and $z = h_s$, where we will refer to h_s as the *surface layer height*. The parameter h_s is chosen for the application of interest. For example, if we are interested in surface damage, it might suffice to only examine the flow in the layer with $h_s = 1$ meter, whereas structural engineers might be interested in multistory buildings, and would necessarily use a larger value for this parameter.

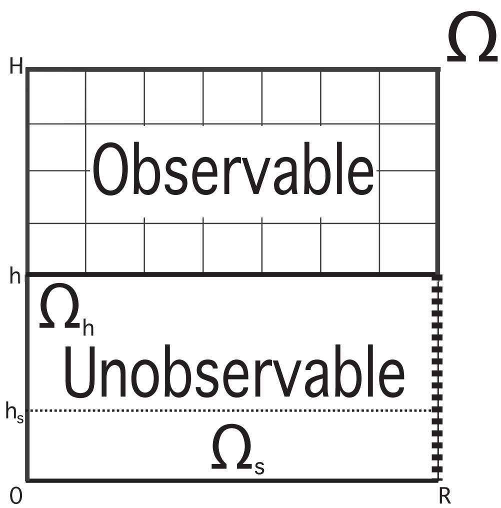

Fig. 1. Schematic of problem domain. The outer radial boundary is dashed to represent the unknown boundary condition.