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Variable separation solutions to the coupled integrable dispersionless equations

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ABSTRACT

In this paper, the variable separation solutions of the coupled integrable dispersionless equations are derived. These solutions include two arbitrary functions and they are more general than the results presented before. Furthermore, some coherent structures of the physical quantity including decaying bell solitary, single dromion, soliton-type breather, peakon-type breather and loop-type breather are constructed.

1. Introduction

During the past several years, the study of coupled dispersionless integrable systems has been an active field of investigations in theoretical and mathematical physics [1–9]. As a special case of a more general set of integrable equations, Konno and Oono presented a new coupled integrable dispersionless (CID) equations

$$\begin{cases} q_t + 2rr_x = 0, \\ r_{xt} - 2qr = 0, \end{cases} \tag{1}$$

where q, r are both functions of x, t and the subscripts denote derivatives. They obtained the one-soliton solution for dark-type and bright-type for this system via inverse scattering method and proposed a generalized inverse scheme from the group theoretical point of view [10,11]. These coupled integrable dispersionless equations describe the current-fed string within an external magnetic field. The detailed physical application of the system was stated in [12]. Later, Kotlyarov proved that this integrable model was gauge equivalent to the sine–Gordon and Pohlmeyer–Lund–Regge model [13]. Again, Konno and Kakuhata obtained the soliton solutions for growing, decaying and stationary solitons [14]. Interactions among the solitary waves and their properties were also considered by them. Recently, by virtue of the multi-linear variable separation approach, Shen derived variable separate solutions of the system [15] and Zhang et al. presented a common formula with some arbitrary functions to describe suitable physical quantities for some $(1+1)$-dimensional models such as the coupled integrable dispersionless equations [16]. Furthermore, we modified the bilinear equations obtained by Alagesan [17] and conjectured their N-soliton solutions [18]. In another paper [19], Alagesan derived one soliton solution of the equations using the corresponding linear eigenvalue problem.

In this paper, we obtain variable separate solutions to the system by means of singular manifold method [20–25] and a direct ansatz technique. These solutions include two arbitrary functions and they are more general than the results

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presented before. The organization of this paper is as follows. In Section 2, the general variable separation solutions for equations under consideration are obtained by the singular manifold method. In Section 3, some localized coherent structures are investigated. The last section is devoted to conclusion and discussion.

2. Variable separate solutions to CID equations

Following the method introduced by Weiss, Tabor and Carnevale (WTC), one can easily prove that CID equations pass the Painlevé test for PDEs and there are enough arbitrary coefficients in the expansions

$$\begin{cases} q = q_0(x,t)\phi^{-2} + q_1(x,t)\phi^{-1} + q_2(x,t) + q_3(x,t)\phi + \cdots, \\ r = r_0(x,t)\phi^{-1} + r_1(x,t) + r_2(x,t)\phi + \cdots. \end{cases} \tag{2}$$

Here $\phi = \phi(x,t)$ is the non-characteristic singular manifold. According to the singular manifold method, we truncate the Painlevé expansions at the constant level term as

$$\begin{cases} q = q_0(x,t)\phi^{-2} + q_1(x,t)\phi^{-1} + Q(x,t), \\ r = r_0(x,t)\phi^{-1} + R(x,t). \end{cases} \tag{3}$$

Substituting Eq. (3) into Eq. (1) and arranging the coefficients at each order of ϕ, we have

$$\phi^0 : \begin{cases} Q_t + 2RR_x = 0, \\ R_{xt} - 2QR = 0. \end{cases} \tag{4}$$

$$\phi^1 : \begin{cases} q_{1t} + 2r_0R_x + 2r_{0x}R = 0, \\ r_{0xt} - 2q_1R - 2r_0Q = 0. \end{cases} \tag{5}$$

$$\phi^2 : \begin{cases} q_{0t} - q_1\phi_t + 2r_0r_{0x} - 2r_0R\phi_x = 0, \\ r_{0t}\phi_x + r_{0x}\phi_t + r_0\phi_{xt} + 2q_0R + 2q_1r_0 = 0. \end{cases} \tag{6}$$

$$\phi^3 : \begin{cases} q_0\phi_t + r_0^2\phi_x = 0, \\ q_0 - \phi_t\phi_x = 0. \end{cases} \tag{7}$$

Eq. (4) shows that $Q(x,t)$ and $R(x,t)$ solve Eq. (1). Therefore, Eq. (3) constitutes an auto-Bäcklund transformation for CID equations and can be used to generate new solutions. Eq. (7) implies that $q_0 = \phi_t\phi_x$ and $r_0^2 = -\phi_t^2$. Without loss of generality we take $r_0 = I\phi_t$ with I denotes the imaginary unit. Substituting the expressions of q_0 and r_0 into Eq. (6), the compatibility condition requires $q_1 = -\phi_{xt}$ and $R = -\frac{I\phi_{tt}}{2\phi_t}$. Based on these results, from Eq. (5) we solve $Q = \frac{\phi_t\phi_{xtt} - \phi_{tt}\phi_{xt}}{2\phi_t^2}$. Substituting the explicit expressions of R and Q into Eq. (4) leads to the unique singular manifold equation (SME)

$$\phi_t^2\phi_{xttt} - \phi_t\phi_{ttt}\phi_{xt} - 3\phi_{tt}\phi_t\phi_{xtt} + 3\phi_{tt}^2\phi_{xt} = 0. \tag{8}$$

Therefore, if one can solve the above PDE of $\phi(x,t)$, then the solutions to original system are easily obtained by the truncation Eq. (3) as

$$\begin{cases} q = \frac{\phi_t\phi_x}{\phi^2} - \frac{\phi_{xt}}{\phi} + \frac{\phi_t\phi_{xtt} - \phi_{tt}\phi_{xt}}{2\phi_t^2}, \\ r = I\left(\frac{\phi_t}{\phi} - \frac{\phi_{tt}}{2\phi_t}\right). \end{cases} \tag{9}$$

This is the key point of singular manifold method. Generally speaking, it is very difficult to solve SME completely and some assumptions are needed.

Let

$$\phi(x,t) = \alpha + \beta f(x) + \gamma g(t) + \delta f(x)g(t), \tag{10}$$

where $\alpha, \beta, \gamma, \delta$ are arbitrary constants which cannot equal to zero simultaneously, and $f(x), g(t)$ are two arbitrary functions. Although the arbitrary constants can be included in the functions $f(x)$ and $g(t)$ when they are not equal to zero, just as every vector in a vector space can be represented as a linear combination of basis vectors, it is more convenient here to take the singular manifold as a linear combination of the two arbitrary functions. We find that SME is satisfied automatically under this assumption. It means that the ansatz Eq. (10) makes sense and we get a particular solution which is used as seed solution in the auto-Bäcklund transformation as follows

$$\begin{cases} Q = \frac{\phi_t\phi_{xtt} - \phi_{tt}\phi_{xt}}{2\phi_t^2} = 0, \\ R = -\frac{I\phi_{tt}}{2\phi_t} = -\frac{Ig_{tt}}{2g_t}. \end{cases} \tag{11}$$

Furthermore, we also get the general variable separation solutions to original CID equations following Eq. (9) as