Applied Mathematics and Computation 235 (2014) 364–376

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Imaging wave-penetrable objects in a finite depth ocean

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ARTICLE INFO

Keywords:
Finite depth ocean
Inhomogeneous media
Direct sampling method

ABSTRACT

We extend the direct sampling method proposed in Ito et al. (2012) [13] to image a wave-penetrable inhomogeneous medium in a 3D waveguide. Incidences and receivers are available only on part of the surface of a cylinder. The proposed method is basically direct and does not involve any matrix inversions or optimizations, thus computationally very cheap and efficient. Numerical simulations are presented to show the feasibility and effectiveness of the method for acoustic detection in a 3D waveguide. The method is applicable with a few scattered fields corresponding to only one or two incident waves, and is very robust against noise.

1. Introduction

The direct and inverse scattering problems of underwater acoustics have received much attention in recent years, see, e.g., [1,2,4,5,7,14,15,17,18] and the references therein. One of the popular models used for acoustic waves in a finite depth ocean is the waveguide bounded by two parallel planes. Because of the geometric structure, the inverse scattering problems in a parallel waveguide are much harder than similar problems in a homogeneous space. Due to the presence of two boundaries of the waveguide, only a finite number of wave modes can propagate in long distance, while the other modes decay exponentially as a function of distance. This phenomenon increases the ill-posedness of the inverse problem considerably. Assume that the ocean has a pressure released surface and a rigid bottom, we can pose a Dirichlet condition on one of the plane and a Neumann condition on the other. Based on this model, the exact and asymptotic representations of the sound field in a stratified shallow ocean was obtained in [3]. Adding a scatterer to the stratified model, a series of studies have been carried out for the direct and inverse scattering of acoustic waves by obstacles in a waveguide with plane boundaries as well as in an ocean under different sediment settings. We refer readers to [9–11,6,8,12,16,19,20] for more details.

In this paper we extend the direct sampling method proposed in Ito et al. [13] for imaging a wave-penetrable inhomogeneous medium in a 3D finite depth ocean. The method is based on a scattering analysis and involves only computing the inner product of the measured scattered field u^s with fundamental solutions located at the sampling points over the measurement curve/surface. The method is basically direct and does not involve any matrix inversions or optimizations. Our numerical experiments indicate that it can provide an accurate and reliable estimate of the support of unknown scatterers, even in the presence of a fairly large amount of noise in the measured data. Consequently, it can be regarded as an effective but simple computational alternative to existing tools for locating a reliable approximate positions of the unknown obstacles, or for generating an initial sampling region for the use in a more refined or computationally more demanding optimization-type algorithm.

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[1] The work of this author was substantially supported by Hong Kong RGC Grant (Project 404611).

http://dx.doi.org/10.1016/j.amc.2014.02.048
0096-3003/

The new method applies a sampling-type technique, and resembles the linear sampling-type methods (LSM) [5,6,17,20], but it differs significantly from these existing techniques. Firstly, it does not perform any matrix inversions, or solves ill-posed linear integral equations, thus is computationally cheap. Secondly, the novel method requires only a few (e.g., one or two) incident waves for reconstructing the locations of scatterers/inhomogeneities, whereas the others usually require the data from sufficiently many incidents in order to acquire a reasonable reconstruction. Lastly, the new method is highly tolerant to noise.

The paper is organized as follows. In Section 2, the direct scattering problem for the 3D waveguide is presented, along with some useful notations, properties and identities. In Section 3, we generate an iterative method to solve the direct scattering problem of the 3D waveguide. Section 4 describes the mathematical motivation of the extended direct sampling method using the near-field data and proposes a new indicator function. Section 5 provides extensive numerical experiments to evaluate the performance of the novel indicator function by the near-field data from scatterers. Finally, some concluding remarks are stated in Section 6.

2. The direct scattering problem for the 3D waveguide

In this section, we describe the direct scattering problem of our interest. Consider a three-dimensional waveguide $\mathbb{R}^3_h = \mathbb{R}^2 \times (0,h)$ for $h > 0$. The third coordinate axis is singled out as the one orthogonal to the waveguide, so we shall write

$$x = (x_1, x_2, x_3)^\top = (\tilde{x}, x_3)^\top \quad \forall x \in \mathbb{R}^3.$$

The upper and lower boundaries of the waveguide are denoted respectively by

$$\Gamma^+ := \{x \in \mathbb{R}^3; x_3 = h\} \quad \text{and} \quad \Gamma^- := \{x \in \mathbb{R}^3; x_3 = 0\}.$$

A bounded and wave-penetrable scatterer D is assumed to be compactly contained in the waveguide. The part of the waveguide not occupied by $\overline{D}$ is denoted by $\Omega := \mathbb{R}^3_h \setminus \overline{D}$, which is assumed to be connected. Let us point out here that we shall often work later in the bounded domain $\Omega_R := \{x \in \Omega : |\tilde{x}|^2 < R^2\}$, where the radius R is assumed to be large enough such that $1 + |\tilde{x}|^2 < R^2$ for all $(\tilde{x}, x_3) \in D$. This implies that $\overline{D}$ is contained in the interior of Ω_R. The surface of the cylinder

$$\Gamma_R := \{x \in \Omega; |\tilde{x}|^2 = R^2\}$$

denotes the boundary of Ω_R that is contained in Ω. The two other parts of $\partial\Omega_R$, contained in the upper and lower boundaries of the waveguide, are denoted by

$$\Gamma^+_R := \{x \in \mathbb{R}^3; |\tilde{x}| < R,\ x_3 = h\} \quad \text{and} \quad \Gamma^-_R := \{x \in \mathbb{R}^3; |\tilde{x}| < R,\ x_3 = 0\}.$$

Fig. 1 shows a diagram of the waveguide's geometry.

The direct scattering of acoustic wave in a parallel waveguide with a wave-penetrable medium is modeled by the Helmholtz equation

$$\Delta u + k^2 n(x) u = f(x) \ \text{ in } \ \mathbb{R}^3_h, \tag{2.1}$$

where $u = u^i + u^s$, formed by the incident wave u^i and its corresponding scattered wave u^s. The sound-hard and sound-soft boundary conditions are imposed on the lower and upper boundaries of the waveguide, i.e.,

$$u = 0 \ \text{ on } \ \Gamma^- \quad \text{and} \quad \frac{\partial u}{\partial x_3} = 0 \ \text{ on } \ \Gamma^+. \tag{2.2}$$

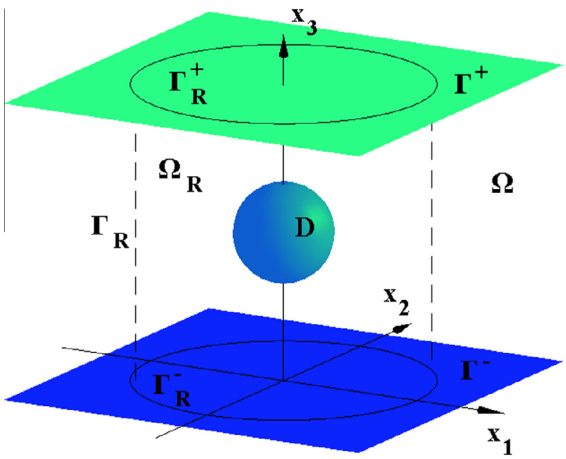

Fig. 1. Geometrical illustration of the waveguide.