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Some identities of symmetry for the generalized q-Euler polynomials

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ABSTRACT

By the symmetric properties of Drichlet's type multiple $q-l$-function, we establish various identities concerning the generalized higher-order q-Euler polynomials. Furthermore, we give some interesting relationship between the power sums and the generalized higher-order q-Euler polynomials.

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1. Introduction

Let χ be a Dirichlet character with conductor $d \in \mathbb{N}$ with $d \equiv 1 \pmod 2$. As is well known, the generalized higher-order Euler polynomials are defined by the generating function to be

$$\left(2\sum_{a=0}^{d-1}\frac{(-1)^a\chi(a)e^{at}}{e^{dt}+1}\right)^r e^{xt} = \sum_{n=0}^{\infty} E_{n,\chi}^{(r)}(x)\frac{t^n}{n!}. \tag{1.1}$$

When $x = 0$, $E_{n,\chi}^{(r)} = E_{n,\chi}^{(r)}(0)$ are called the generalized Euler numbers attached to χ of order $r(\in \mathbb{N})$.

For $q \in \mathbb{C}$ with $|q| < 1$, the q-number is defined by $[x]_q = \frac{1-q^x}{1-q}$.

Note that $\lim_{q\to 1}[x]_q = x$. In [7], Kim considered q-extension of generalized higher-order Euler polynomials attached to χ as follows:

$$F_{q,\chi}^{(r)}(t,x) = [2]_q^r \sum_{m_1,\ldots,m_r=0}^{\infty} (-q)^{m_1+\cdots+m_r}\left(\Pi_{i=1}^r\chi(m_i)\right)e^{[m_1+\cdots+m_r+x]_q t} = \sum_{n=0}^{\infty} E_{n,\chi,q}^{(r)}(x)\frac{t^n}{n!}. \tag{1.2}$$

Note that

$$\lim_{q\to 1} F_{q,\chi}^{(r)}(t,x) = \left(2\sum_{a=0}^{d-1}\frac{(-1)^a\chi(a)e^{at}}{e^{dt}+1}\right)^r e^{xt}.$$

For $s \in \mathbb{C}$ and $x \in \mathbb{R}$ with $x \neq 0, -1, -2, \ldots$, Kim defined Dirichlet-type multiple $q-l$-funtion which is given by

$$l_{q,r}(s,x|\chi) = [2]_q^r \sum_{m_1,\ldots,m_r=0}^{\infty} \frac{(-q)^{m_1+\ldots+m_r}\left(\Pi_{i=1}^r\chi(m_i)\right)}{[m_1+\cdots+m_r+x]_q^s} = \frac{1}{\Gamma(s)}\int_0^{\infty} F_{q,\chi}^{(r)}(-t,x)t^{s-1}dt, \quad \text{(see [7])}. \tag{1.3}$$

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Applying the Laurent series and Cauchy residue theorem in (1.2) and (1.3), we get

$$l_{q,r}(-n,x|\chi) = E^{(r)}_{n,\chi,q}(x), \quad \text{where } n \in \mathbb{Z}_{\geqslant 0}. \tag{1.4}$$

When $x = 0$, $E^{(r)}_{n,\chi,q} = E^{(r)}_{n,\chi,q}(0)$ are called the generalized q-Euler numbers attached to χ of order r. From (1.2), we note that

$$E^{(r)}_{n,\chi,q}(x) = \sum_{l=0}^{n}\binom{n}{l} q^{lx}E^{(r)}_{l,\chi,q}[x]_q^{n-l} = \left(q^x E^{(r)}_{\chi,q} + [x]_q\right)^n \tag{1.5}$$

with the usual convention about replacing $\left(E^{(r)}_{\chi,q}\right)^n$ by $E^{(r)}_{n,\chi,q}$, (see [1–13]).

In this paper, we investigate properties of symmetry in two variables related to multiple q—l-function which interpolates generalized higher-order q-Euler polynomials attached to χ at negative integers. From our investigation, we derive identities of symmetry in two variables related to generalized higher-order q-Euler polynomials attached to χ. Recently, several authors have studied q-extensions of Euler polynomials due to Kim (see [1–3,9–13]).

2. Symmetry of q-power sum and the generalized q-Euler polynomials

For $a, b \in \mathbb{N}$ with $a \equiv 1 \pmod 2$ and $b \equiv 1 \pmod 2$, we observe that

$$\frac{1}{[2]^r_{q^a}} l_{q^a,r}\left(s, bx + \frac{b}{a}(j_1 + \cdots + j_r)|\chi\right) = [a]^s_q \sum_{n_1,\ldots,n_r=0}^{\infty} \sum_{i_1,\ldots,i_r=0}^{db-1} \frac{(-1)^{\sum_{l=1}^r (i_l+n_l)} q^{a\sum_{l=1}^r (i_l+bdn_l)} \left(\prod_{l=1}^r \chi(i_l)\right)}{\left[ab\left(x + d\sum_{l=1}^r n_l\right) + b\sum_{l=1}^r j_l + a\sum_{l=1}^r i_l\right]^s_q}. \tag{2.1}$$

From (2.1), we have

$$\begin{aligned}
&\frac{[b]^s_q}{[2]^r_{q^a}} \sum_{j_1,\ldots,j_r=0}^{da-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{b\sum_{l=1}^r j_l} l_{q^a,r}\left(s, bx + \frac{b}{a}\sum_{l=1}^r j_l|\chi\right) \\
&= [a]^s_q [b]^s_q \sum_{i_1,\ldots,i_r=0}^{db-1} \sum_{j_1,\ldots,j_r=0}^{da-1} \sum_{n_1,\ldots,n_r=0}^{\infty} \frac{(-1)^{\sum_{l=1}^r (i_l+n_l+j_l)} \left(\prod_{l=1}^r \chi(j_l)\right)\left(\prod_{l=1}^r \chi(i_l)\right)}{\left[ab\left(x + d\sum_{l=1}^r n_l\right) + \sum_{l=1}^r (bj_l + ai_l)\right]^s_q} \times q^{\sum_{l=1}^r (ai_l + bj_l + abdn_l)}.
\end{aligned} \tag{2.2}$$

By the same method as (2.2), we get

$$\begin{aligned}
&\frac{[a]^s_q}{[2]^r_{q^b}} \sum_{j_1,\ldots,j_r=0}^{db-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{a\sum_{l=1}^r j_l} l_{q^b,r}\left(s, ax + \frac{a}{b}\sum_{l=1}^r j_l|\chi\right) \\
&= [a]^s_q [b]^s_q \sum_{j_1,\ldots,j_r=0}^{db-1} \sum_{i_1,\ldots,i_r=0}^{da-1} \sum_{n_1,\ldots,n_r=0}^{\infty} \frac{(-1)^{\sum_{l=1}^r (i_l+n_l+j_l)} \left(\prod_{l=1}^r \chi(j_l)\chi(i_l)\right)}{\left[ab\left(x + d\sum_{l=1}^r n_l\right) + \sum_{l=1}^r (aj_l + bi_l)\right]^s_q} \times q^{\sum_{l=1}^r (aj_l + bi_l + abdn_l)}.
\end{aligned} \tag{2.3}$$

Therefore, by (2.2) and (2.3), we obtain the following theorem.

Theorem 2.1. *For $a, b \in \mathbb{N}$ with $a \equiv 1 \pmod 2$2 and $b \equiv 1 \pmod 2$, we have*

$$\begin{aligned}
&[2]^r_{q^b}[b]^s_q \sum_{j_1,\ldots,j_r=0}^{da-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{b\sum_{l=1}^r j_l} l_{q^a,r}\left(s, bx + \frac{b}{a}\sum_{l=1}^r j_l|\chi\right) \\
&= [2]^r_{q^a}[a]^s_q \sum_{j_1,\ldots,j_r=0}^{db-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{a\sum_{l=1}^r j_l} l_{q^b,r}\left(s, ax + \frac{a}{b}\sum_{l=1}^r j_l|\chi\right).
\end{aligned}$$

From (1.4) and Theorem 2.1, we obtain the following theorem.

Theorem 2.2. *For $n \geqslant 0$ and $a, b \in \mathbb{N}$ with $a \equiv 1 \pmod 2$ and $b \equiv 1 \pmod 2$, we have*

$$\begin{aligned}
&[2]^r_{q^b}[a]^n_q \sum_{j_1,\ldots,j_r=0}^{da-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{b\sum_{l=1}^r j_l} E^{(r)}_{n,\chi,q^a}\left(bx + \frac{b}{a}\sum_{l=1}^r j_l\right) \\
&= [2]^r_{q^a}[b]^n_q \sum_{j_1,\ldots,j_r=0}^{db-1} (-1)^{\sum_{l=1}^r j_l} \left(\prod_{l=1}^r \chi(j_l)\right) q^{a\sum_{l=1}^r j_l} E^{(r)}_{n,\chi,q^b}\left(ax + \frac{a}{b}\sum_{l=1}^r j_l\right).
\end{aligned}$$

By (1.5), we easily get

$$E^{(r)}_{n,\chi,q}(x+y) = \left(q^{x+y}E^{(r)}_{\chi,q} + [x+y]_q\right)^n = \left(q^{x+y}E^{(r)}_{\chi,q} + q^x[y]_q + [x]_q\right)^n = \sum_{i=0}^{n}\binom{n}{i} q^{ix}\left(q^y E^{(r)}_{\chi,q} + [y]_q\right)^i [x]_q^{n-i} = \sum_{i=0}^{n}\binom{n}{i} q^{xi}E^{(r)}_{i,\chi,q}(y)[x]_q^{n-i}. \tag{2.4}$$

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