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On the strong convergence of an iterative process for asymptotically strict pseudocontractions and equilibrium problems

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ABSTRACT

In this paper, equilibrium and fixed point problems based on hybrid iterative processes are investigated. It is proved that the sequence generated in the purposed iterative process converges strongly to a common element in the fixed point set of an asymptotically strict pseudocontraction and in the solution set of an equilibrium problem in the framework of real Hilbert spaces.

1. Introduction–preliminaries

Throughout this paper, we always assume that H is a real Hilbert space with the inner product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$. Let C be a nonempty closed convex subset of H and F a bifunction of $C\times C$ into $\mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. In this paper, we consider the following equilibrium problem.

$$\text{Find } x\in C \quad \text{such that} \quad F(x,y)\geqslant 0,\quad \forall y\in C. \tag{1.1}$$

The set of such an $x\in C$ is denoted by $EP(F)$, i.e.,

$$EP(F)=\{x\in C: F(x,y)\geqslant 0,\quad \forall y\in C\}.$$

The equilibrium problem, which is known as the Ky Fan inequality, first introduced and studied by Ky Fan [1]. To study the equilibrium problem (1.1), we may assume that F satisfies the following conditions:

(A1) $F(x,x)=0$ for all $x\in C$;
(A2) F is monotone, i.e., $F(x,y)+F(y,x)\leqslant 0$ for all $x,y\in C$;
(A3) for each $x,y,z\in C$,

$$\limsup_{t\downarrow 0} F(tz+(1-t)x,y)\leqslant F(x,y);$$

(A4) for each $x\in C$, $y\mapsto F(x,y)$ is convex and lower semi-continuous.

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The equilibrium problems has been revealed as a very powerful and important tool in the study of nonlinear phenomena. It is well known that the equilibrium problem includes many important problems in nonlinear analysis and optimization such as the Nash equilibrium problem, variational inequalities, complementarity problems, vector optimization problems, fixed point problems, saddle point problems and game theory. The equilibrium problem recently has been studied as an effective and powerful tool for studying a wide class of real world problems which arise in economics, finance, image reconstruction, ecology, transportation, and network; see [2–18] and the references therein.

Let $S : C \to C$ be a mapping. In this paper, we use $F(S)$ to denote the fixed point set of S. Recall the following definitions.

(a) S is said to be nonexpansive if

$$\|Sx - Sy\| \leqslant \|x - y\|, \quad \forall x, y \in C.$$

(b) S is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|S^n x - S^n y\| \leqslant k_n \|x - y\|, \quad \forall x, y \in C, \quad n \geqslant 1.$$

The class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [19] in 1972. It is known that if C is a nonempty bounded closed convex subset of a Hilbert space space H, then every asymptotically nonexpansive self-mapping has a fixed point. Further, the set $F(S)$ of fixed points of S is closed and convex. Since 1972, a host of authors have studied the weak and strong convergence problems of the iterative processes for such a class of mappings.

(c) S is said to be strictly pseudocontractive if there exists a constant $\kappa \in [0, 1)$ such that

$$\|Sx - Sy\|^2 \leqslant \|x - y\|^2 + \kappa\|(I - S)x - (I - S)y\|^2, \quad \forall x, y \in C.$$

For such a case, S is also said to be κ-strict pseudocontraction. The class of strict pseudocontractions is introduced by Brower and Petryshyn [20] in 1967. It is clear that every nonexpansive mapping is a 0-strict pseudocontraction. We also remark that if $\kappa = 1$, then S is said to be pseudocontractive.

(d) S is said to be an asymptotically strict pseudocontraction if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ and a constant $\kappa \in [0, 1)$ such that

$$\|S^n x - S^n y\|^2 \leqslant k_n\|x - y\|^2 + \kappa\|(I - S^n)x - (I - S^n)y\|^2, \quad \forall x, y \in C, \quad n \geqslant 1.$$

For such a case, S is also said to be an asymptotically κ-strict pseudocontraction. The class of asymptotically strict pseudocontractions is introduced by Qihou [21] in 1996. It is clear that every asymptotically nonexpansive mapping is an asymptotical 0-strict pseudocontraction. We also remark that if $\kappa = 1$, then S is said to be an asymptotically pseudocontractive mapping which was introduced by Schu [22] in 1991. Recently, many authors investigated fixed points of asymptotically strict pseudocontractions based on hybrid projection methods; see [23–26] and the references therein.

A closely related subject of current interest is the problem of finding common elements in the fixed point set of nonlinear operators and in the solution set of the equilibrium problem (1.1); see [27–32] and the references therein. The motivation for this subject is mainly due to its possible applications to mathematical modeling of concrete complex problems. Indeed, there are many mathematical modelings in which we often have to use more than one constraint. Solving such problems, we have to obtain some solution which is simultaneously the solution of two or more subproblems or the solution of one subproblem on the solution set of another subproblem.

In 2007, Tada and Takahashi [27] considered a hybrid iterative method for the equilibrium problem (1.1) and a nonexpansive mapping based on the following algorithm

$$\begin{cases} x_0 \in C \text{ chosen arbitrarily}, \\ u_n \in C \quad \text{such that} \quad F(u_n, y) + \frac{1}{r_n}\langle y - u_n,\ u_n - x_n\rangle \geqslant 0, \quad \forall y \in C, \\ w_n = (1 - \alpha_n)x_n + \alpha_n S u_n, \\ C_n = \{z \in H : \|w_n - z\| \leqslant \|x_n - z\|\}, \\ D_n = \{z \in H : \langle x_n - z,\ x - x_n\rangle \geqslant 0\}, \\ x_{n+1} = P_{C_n \cap D_n} x, \end{cases}$$

where S is a nonexpansive mapping, $\{\alpha_n\} \subset [a, 1]$, for some $a \in (0, 1)$ and $\{r_n\} \subset (0, \infty)$ satisfies $\liminf_{n\to\infty} r_n > 0$. They proved that the sequence $\{x_n\}$ converges strongly to $P_{F(S)\cap EP(F)}(x)$.

In 2008, Kim and Xu [24] further considered the class of asymptotically strict pseudocontractions based on the hybrid projection method in a real Hilbert space. To be more precise, they investigated the following algorithm:

$$\begin{cases} x_0 \in C \text{ chosen arbitrarily}, \\ y_n = \alpha_n x_n + (1 - \alpha_n)S^n x_n, \\ C_n = \left\{w \in C : \|y_n - w\|^2 \leqslant \|x_n - w\|^2 + [\kappa - \alpha_n(1 - \alpha_n)]\|x_n - S^n x_n\|^2 + \theta_n\right\}, \\ D_n = \{w \in C : \langle x_n - w,\ x_0 - x_n\rangle \geqslant 0\}, \\ x_{n+1} = P_{C_n \cap D_n} x_0, \quad n \geqslant 0, \end{cases}$$