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Adaptive backstepping synchronization between chaotic systems with unknown Lipschitz constant

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ABSTRACT

The error between two nonlinear terms is a key point of many synchronization problems, however, the Lipschitz constant of the nonlinear term is not always easy to calculate for the stability analysis of the controlled error system, thus the nonlinear systems with unknown parameters and unknown Lipschitz constant is considered in this paper. Their scalar synchronous controller is proposed based on the thought of backstepping design. Without the need to evaluate the invariant set and calculate the Lipschitz constant, an assistant adaptive estimator is designed for the Lipschitz constant. What's more, as a problem solving skill, two different estimators are used on the same unknown parameter. Finally, the synchronization control for both chaotic autonomous Van der Pol–Duffing (ADVP) systems and chaotic Genesio systems with unknown parameters are given as examples to verify the control effect.

1. Introduction

Backstepping design was first given in [1]. With the development of nonlinear control, it has become a systematic and effective method to resolve the control and synchronization problems [2–11]. The basic steps of backstepping control can be simply described as decomposing the system into several lower dimensional subsystems, designing partial Lyapunov functions and virtual controllers, and finally getting the scalar controller, and hence backstepping controller is especially suitable for chaos control because irregular oscillations often exist in chaotic systems and the controller is not convenient to be added on all subsystems.

Backstepping design has been widely used in the synchronization problems. In [12–14], synchronization of chaotic autonomous Van der Pol–Duffing (ADVP) system, chaotic Josephson junction and hyperchaotic Liu system was considered, without any uncertainty considered. And in [15,16], systems with unknown or uncertain sections and external disturbances were studied. In [15], backstepping design was combined with cerebellar model articulation controller, which could synchronize one system to the other with disturbances and uncertainties. In [16], the Gaussian radial basis function neural network was lead in the controller to approximate the unknown nonlinear terms.

In fact, an important problem in synchronization control is the treatment of the error between two nonlinear terms like $F(e) = f(x) - f(y)$, where $f(\cdot)$ is the nonlinear link of the master system and the slave system. $F(e)$ and $f(x)$ were often substituted into the controller or approximated as a whole in many literatures, such as [17,16]. In [18], solutions to the stabilization

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problems of nonlinear discrete-time system were provided in the form of linear matrix inequalities (LMIs), where the unknown nonlinearities were required to satisfy Lipschitz conditions in the state and delayed-state. In [19], Lipschitz constant of $f(\cdot)$ was calculated, but it is a complex process. Actually, Lipschitz constants are not easy to calculate in most cases. Hence, in this paper we will adopt a straight method: adaptive estimation to the Lipschitz constant of $f(\cdot)$, and we will consider the synchronization of the nonlinear system in the form of the combination of a linear subsystem and a scalar nonlinear subsystem, with unknown Lipschitz constant and multiple unknown parameters.

Then the synchronous effect of the controller will be validated by both chaotic ADVP systems and chaotic Genesio systems. Early, ADVP system appeared in [20,21]. Its form is equivalent to Chua's autonomous circuit but with a cubic nonlinear element. Its dynamic behavior was discussed in [22], which contains equilibrium point, periodic orbit and bifurcation. Recently, modified ADVP model was given in [12] with further analysis on the stability conditions of its equilibrium point and the existence of the period solution. In [22], the modified ADVP's abundant dynamic behaviors were discussed, such as the local codimension one, two and three bifurcations. In addition, chaos control of ADVP model was achieved in [23–25], and the synchronization between certain ADVP systems was realized in [2,12] using back stepping design.

Chaotic Genesio system (CGS) was proposed in 1992 [26], which consists of three simple ordinary differential equations with a simple nonlinear term. In recent years, increasing attention has been paid on this system. In [27], the high-accuracy solution of CGS was given, which can be treated as an accurate method of analyzing its dynamic behavior. In [28], a design method for the control of CGS was proposed based on the combination of Lyapunov stability theorem and LMI optimization approach. The chaos control and chaos synchronization of the CGS were also discussed in [29–31]. The backstepping approach was also used to solve these problems in [32,33]. However, the discussion on the case of unknown Lipschitz constant is relatively rare. This problem will be solved in this paper.

This paper is organized as follows. In Section 2, we will give the system's models and the problem to be settled. In Section 3, the design of adaptive backstepping synchronous controller will be expounded in detail. In Section 4, chaos synchronization of chaotic ADVP system and CGS will be provided as examples to verify the control effect.

2. Systems' description

For common adaptability of the following procedure, we consider the following response system and driving system:

$$\begin{cases} \dot{\bar{x}} = \lambda f(\bar{x}) + (\beta + \lambda_\beta)\bar{x} + (\gamma + \lambda_\gamma)\bar{y} + u, \\ \dot{\bar{y}} = (A + \lambda_A)\bar{y} + b\bar{x}, \end{cases} \tag{1}$$

$$\begin{cases} \dot{x} = \lambda f(x) + (\beta + \lambda_\beta)x + (\gamma + \lambda_\gamma)y, \\ \dot{y} = (A + \lambda_A)y + bx, \end{cases} \tag{2}$$

where $\bar{x},\ x \in \mathcal{R},\ \bar{y},\ y \in \mathcal{R}^n$ are the state variables, $u \in \mathcal{R}$ is a scalar controller, $A \in \mathcal{R}^{n\times n},\ b \in \mathcal{R}^n$ are the coefficient matrix and coefficient vector. Without loss of generality, let $\lambda > 0$. Besides, the nonlinear mapping $f(\cdot)$ satisfies $|f(\bar{x}) - f(x)| \leqslant L|\bar{x} - x|$, and Lipschitz constant L is unknown. Other factors are scalars, matrices and vectors with proper dimensions, of which $\lambda,\ \lambda_\beta,\ \lambda_\gamma,\ \lambda_A$ are unknown, and $\lambda_A = \sum_{i=1}^m \alpha_i A_i$, where $A_i \in \mathcal{R}^{n\times n}$ $(i = 1, 2, \ldots, m)$ are known constant matrices, α_i $(i = 1, 2, \ldots, m)$ is unknown weighting constants.

Let $e_x = \bar{x} - x$, $e_w = \bar{y} - y$, and $e = \left[e_x^{\mathrm{T}}, e_w^{\mathrm{T}}\right]^{\mathrm{T}}$, then the error system is

$$\begin{cases} \dot{e}_x = \lambda F(e_x) + (\beta + \lambda_\beta)e_x + (\gamma + \lambda_\gamma)e_w + u, & (a) \\ \dot{e}_w = (A + \lambda_A)e_w + be_x, & (b) \end{cases} \tag{3}$$

where $e_x \in \mathcal{R}$, $e_w \in \mathcal{R}^n$ are state variables, and let $F(e_x) = f(\bar{x}) - f(x)$. The object is to design the controller u to make system (3) asymptotically stable.

3. Synchronization control based on adaptive backstepping approach

3.1. Virtual controller and system transformation

Let the virtual controller of Eq. (3b) be $e_x(e_w) = \left(k^{\mathrm{T}} - c^{\mathrm{T}}\sum_{i=1}^m \hat{\alpha}_i A_i\right)e_w$, where $\hat{\alpha}_i$ is the estimation of α_i, $k \in \mathcal{R}^n$ is an undetermined constant vector. Denote $\bar{A} = A + bk^{\mathrm{T}}$, suppose that $c \in \mathcal{R}^n$ satisfies $c^{\mathrm{T}}b = 1$. Select $\dot{\hat{\alpha}}_i = e_w^{\mathrm{T}} PA_i e_w$ $(i = 1, 2, \ldots, m)$ as the adaptive law of $\hat{\alpha}_i$, and define a partial Lyapunov function $V_1 = e_w^{\mathrm{T}} P e_w + \sum_{i=1}^m \tilde{\alpha}_i^2$, where $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i$ and P is a positive definite symmetric matrix. The time derivative of V_1 is given by

$$\dot{V}_1 = 2e_w^{\mathrm{T}} P\dot{e}_w + 2\sum_{i=1}^m \tilde{\alpha}_i \dot{\tilde{\alpha}}_i = 2e_w^{\mathrm{T}} P\left[Ae_w + bk^{\mathrm{T}} e_w + \sum_{i=1}^m \alpha_i A_i e_w - bc^{\mathrm{T}}\sum_{i=1}^m \hat{\alpha}_i A_i e_w\right] - 2\sum_{i=1}^m \tilde{\alpha}_i e_w^{\mathrm{T}} PA_i e_w = e_w^{\mathrm{T}}(\bar{A}^{\mathrm{T}} P + P\bar{A})e_w. \tag{4}$$

According to above virtual controller $e_x(e_w)$, we define an invertible coordinate transformation $\tilde{e}_x = e_x - \left(k^{\mathrm{T}} - c^{\mathrm{T}}\sum_{i=1}^m \hat{\alpha}_i A_i\right)e_w$, $e_w = e_w$, and Eq. (3) is transformed into