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Approximate controllability of semilinear nonlocal fractional differential systems via an approximating method

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ABSTRACT

We study the control system governed by a class of abstract nonlocal fractional differential equations. By using the fractional calculus and approximating technique, we give the approximate problem of the control system and get the compactness of approximate solutions set. Then new sufficient conditions for the approximate controllability of the control system are obtained when the compactness conditions or Lipschitz conditions for the nonlocal function are not required.

1. Introduction

This paper is concerned with the approximate controllability of the following fractional differential equations with nonlocal conditions

$$\begin{cases} D^q x(t) = Ax(t) + f(t, x(t)) + Bu(t), \quad t \in J = [0, b], \\ x(0) + g(x) = x_0, \end{cases} \tag{1}$$

where the state variable $x(\cdot)$ takes values in the Hilbert space $X; D^q$ is the Caputo fractional derivative of order q with $0 < q \leqslant 1$; $A : D(A) \subset X \to X$ is the infinitesimal generator of a strongly continuous semigroup $T(t)$ on a Hilbert space X; the control function $u(\cdot)$ is given in $L^2(J, U), U$ is a Hilbert space; B is a bounded linear operator from U into $X; f$ and g are appropriate continuous functions to be specified later.

Fractional differential equations are considered as useful models for describing real world problems, which cannot be described using classical integer order differential equations, see for instance [1–3]. El-Borai [4] studied fundamental solution of fractional evolution equations in a Banach space. By using Laplace transformation, Wang and Zhou [5] give the concept of mild solutions to a class of fractional evolution equations. Balachandran and Park [6] prove the existence of solutions of fractional nonlocal evolution equations by using fractional calculus and fixed point theorems. The fractional differential equations with nonlocal conditions are also considered by [7–10], as nonlocal problems have better effects in applications than the classical ones $x(0) = x_0$. The initial work of nonlocal condition $x(0) + g(x) = x_0$ applied to abstract differential equation is due to Byszewski and Lakshmikantham [11]. Then different topics on the existence and qualitative properties of solutions are considered. For more details we refer the reader to [12–15] and references therein.

On the other hand, the issue of controllability plays an important role in control theory and engineering. The problem of controllability of various kinds of differential, integrodifferential equations and impulsive differential equations are studied,

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see [16,17]. Now this issue has been developed into three directions. The first one is concerned with exact controllability, which steer the system to arbitrary final state. The main approach is to convert the controllability problem into a fixed point problem with assumption that the controllability operator has an induced inverse, see [18–20]. Hernández and O'Regan [21] yet have shown that some papers on exact controllability of abstract control systems contain a similar technical error when the compactness of semigroup $T(t)$ and other hypotheses are satisfied, i.e., in this case the application of controllability results is restricted to the finite-dimensional space. Recently, the method of measure of noncompactness is adopted to overcome this problem and get the exact controllability under noncompact semigroup, see [22,23]. The second direction is null controllability, i.e., for any given initial state x_0, there exists a control u such that the system can be steered to final state of zero point. It can be seen as a special case of exact controllability in some way. The third direction is concerned with approximate controllability, which means that the system can be steered to arbitrary small neighborhood of final state, see [24,25]. As exact controllability holds true only in finite-dimensional space under the compact assumptions to semigroup $T(t)$ or the operator B, it is important to study this weaker concept of controllability in abstract spaces.

Recently, the approximate controllability of fractional order differential systems have been considered by Kumar et al. [26] and Sakthivel et al. [27,28], where the operator semigroup is supposed to be compact and the probability density functions are introduced to define the mild solution. Sakthivel et al. [27], Mahmudov [29] studied the nonlocal fractional differential system (1) when the nonlocal function g is supposed to be Lipschitz continuous or compact in different frameworks. When some fixed point theorems such as Banach's and Schauder's fixed point theorems are applied to get a fixed point for solution operators, functions f and g are often supposed to be compact or Lipschitz continuous. However, this property is not often satisfied in practical applications. One purpose of this article is to investigate the approximate controllability of system (1) without the Lipschitz continuous or compact assumptions on the nonlocal item g. Actually, g is only assumed to be continuous and is completely determined on $[\delta, b]$ for some small $\delta > 0$. Meanwhile, in order to get the existence of solutions to control system (1), we construct the approximate problem of system (1) (see formula (9)) and get the compactness of the approximate solutions set (see formula (11)). It is different from the usual approach that the fixed point theorem is applied directly to the concerned solution operator. So our results can be regarded as extension and development of the existing conclusions.

This article is organized in the following way. In Section 2, we recall some definitions on Caputo fractional derivatives and mild solutions to Eq. (1). In Section 3, the existence result of mild solutions is given by operator semigroup theory and approximating method. In Section 4, sufficient conditions for the approximate controllability are proved. Finally, an example is given to illustrate the application of our results.

2. Preliminaries

Throughout this paper, let $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}_+$ be the set of positive integers, real numbers and positive real numbers, respectively. We denote by X a Hilbert space with norm $\|\cdot\|$, $C([0,b];\ X)$ the space of all $X-$valued continuous functions on $[0,b]$, $B(X)$ the space of all bounded linear operators from X to X with the norm $\|Q\|_{B(X)} = \sup\{\|Q(x)\| : \|x\| = 1\}$, where $Q \in B(X)$. In this paper, let A be the infinitesimal generator of C_0 semigroup $\{T(t)\}_{t\geqslant 0}$ of uniformly bounded liner operators on X. Clearly, $M = \sup_{t\in[0,\infty)}\|T(t)\| < \infty$.

Now we recall some definitions and results on fractional derivative and fractional differential equations.

Definition 2.1 [30]. The Riemann–Liouville fractional integral of a function $f \in L^1([0,\infty];\mathbb{R}_+)$ of order $\alpha \in \mathbb{R}_+$ is defined by

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(s)\,\mathrm{d}s,$$

where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2 [30]. The Riemann–Liouville fractional order derivative of order $\alpha \in \mathbb{R}_+$ of a function f given on the interval $[0,+\infty)$ is defined by

$$^{R-L}D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\frac{\mathrm{d}^n}{\mathrm{d}t^n}\int_0^t (t-s)^{n-\alpha-1} f(s)\,\mathrm{d}s,$$

where $\alpha \in (n-1,n),\ n \in \mathbb{N}$.

Definition 2.3 [30]. The Caputo fractional order derivative of order $\alpha \in \mathbb{R}_+$ of a function f given on the interval $[0,+\infty)$ is defined by

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s)\,\mathrm{d}s,$$

where $\alpha \in (n-1,n),\ n \in \mathbb{N}$.