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A new non-interior continuation method for solving the second-order cone complementarity problem ☆

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ABSTRACT

In this paper, based on a symmetrically perturbed smoothing Fischer–Burmeister function, a non-interior continuation method is proposed for solving the second-order cone complementarity problem (SOCCP). The proposed algorithm solves only one linear system of equations and performs only one line search at each iteration. Under monotonicity, it is shown that our algorithm is globally and locally superlinearly convergent without requiring strict complementarity assumption at the SOCCP solution. Furthermore, the proposed algorithm has local quadratic convergence under mild conditions. Some numerical results are reported which indicate the effectiveness of the proposed algorithm.

1. Introduction

In this paper, we consider the following second-order cone complementarity problem (SOCCP):

$$\text{Find } (x,y) \in \mathcal{R}^n \times \mathcal{R}^n, \quad \text{such that } x \in \mathcal{K},\ y \in \mathcal{K},\ x^{\mathrm{T}}y = 0,\ y = F(x), \tag{1.1}$$

where $F : \mathcal{R}^n \to \mathcal{R}^n$ is a continuously differentiable function, and $\mathcal{K} \subset \mathcal{R}^n$ is the Cartesian product of second-order cones, that is,

$$\mathcal{K} = \mathcal{K}^{n_1} \times \cdots \times \mathcal{K}^{n_r}$$

with $r, n_1, \ldots, n_r \geqslant 1$ and $n = \sum_{i=1}^r n_i$, and $\mathcal{K}^{n_i}$ being the n_i-dimensional second-order cone (SOC) defined by

$$\mathcal{K}^{n_i} := \{(x_1, \bar{x}^{\mathrm{T}})^{\mathrm{T}} \in \mathcal{R} \times \mathcal{R}^{n_i - 1} : x_1 \geqslant \|\bar{x}\|\}. \tag{1.2}$$

Here and below, $\|\cdot\|$ denotes the 2-norm defined by $\|x\| = \sqrt{x^{\mathrm{T}}x}$ for a vector x. If $n_i = 1$, then $\mathcal{K}^1$ is the set of nonnegative reals $\mathcal{R}_+$ (the nonnegative orthant in $\mathcal{R}$). Notice that the complementarity condition on $\mathcal{K} = \mathcal{K}^{n_1} \times \cdots \times \mathcal{K}^{n_r}$ can be decomposed into complementarity conditions on each $\mathcal{K}^{n_i} (i = 1, \ldots, r)$, that is,

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$$x \in \mathcal{K}, \quad y \in \mathcal{K}, \quad x^{\mathrm{T}}y = 0 \Longleftrightarrow x^i \in \mathcal{K}^{n_i}, \quad y^i \in \mathcal{K}^{n_i}, \quad (x^i)^{\mathrm{T}} y^i = 0, \quad i = 1, \ldots, r. \tag{1.3}$$

In the following analysis, for simplicity, we assume that $\mathcal{K} = \mathcal{K}^n$. In view of (1.3), our analysis can be extended to the general case $\mathcal{K}$ in a straightforward manner.

In the last few years, the SOCCP has attracted a lot of attention (e.g., [2,4–6,11,12,22–24]). One motivation to study the SOCCP is its wide applications in many fields (e.g., [13,17]). In addition, the SOCCP includes the well-known nonlinear complementarity problem (NCP) as a special case, corresponding to $n_i = 1, \ i = 1, \ldots, r$. The Karush–Kuhn–Tucker (KKT) conditions for any second-order cone programming (SOCP) can also be written in the form of the SOCCP. A number of methods for solving the SOCCP have been proposed such as the smoothing-regularization method [12], derivative-free descent method [23], damped Gauss–Newton method [22]. In addition, many researchers have worked on interior-point method (IPMs) for the SOCP and the SOCCP and achieved plentiful and beautiful results (e.g., [20,32,33]).

Recently, great attention has been paid to non-interior continuation methods (smoothing Newton methods) partially due to their encouraging convergent properties and numerical results. Many non-interior continuation methods were proposed for solving some mathematical programming problems. However, in order to obtain the local superlinear (quadratic) convergence, some algorithms (e.g., [3,25]) depend strongly on the assumptions of uniform nonsingularity and strict complementarity conditions. Lastly, a class of new smoothing Newton methods was proposed for the NCP and box constrained variational inequalities by Qi et al. [26]. The Qi–Sun–Zhou method [26] was shown to be locally superlinearly/quadratically convergent without strict complementarity. Due to its simplicity and weaker assumptions imposed on smoothing functions, the Qi–Sun–Zhou method [26] has been further studied for the SOCP (e.g., [8–10,29,30]). Tang et al. [31] proposed a smoothing Newton method for solving the SOCCP by modifying and extending the Qi–Sun–Zhou method [26] and established the local quadratic convergence of the algorithm without strict complementarity. It is noted that smoothing functions play an important role in designing and analyzing non-interior continuation methods. Some smoothing functions have been proposed by smoothing the Fischer–Burmeister function (e.g., [8,10,18,29,30]) and the natural residual function (e.g., [9,31]).

In this paper, based on a symmetrically perturbed smoothing Fischer–Burmeister function, we propose a non-interior continuation method for solving the SOCCP by modifying and extending the Qi–Sun–Zhou method [26]. In our method, we adopt a new merit function that is different from the ones used in the existing methods (e.g., [4,6,31]). In addition, besides the classical step length, we propose another new step length. Under monotonicity, the method proposed in this paper possesses the following nice properties. (a) It is well-defined and a solution of the SOCCP can be obtained from any accumulation point of the iteration sequence generated by this method. (b) If the solution set of the SOCCP is nonempty and bounded, then the generated sequence is bounded and hence it has at least one accumulation. (c) Just as the Qi–Sun–Zhou method [26], it can be initialed from an arbitrary point, and needs only to solve one system of linear equations and to do one line search at each iteration. (d) If an accumulation point of the iteration sequence satisfies a nonsingularity assumption, then the whole iteration sequence converges to the accumulation point globally and superlinearly without strict complementarity. Moreover, if the Jacobian of F is Lipschitz continuous on $\mathcal{R}^n$, then the iteration sequence converges quadratically. We also report some numerical results, which indicate that the proposed non-interior continuation method is quite effective for solving the SOCCP.

The paper is organized as follows. In Section 2, we introduce some preliminaries to be used in the subsequent sections. In Section 3, we present a smoothing function and give its properties. In Section 4, we propose a non-interior continuation method for solving the SOCCP and show the well-definedness of the algorithm. The global convergence and local convergence of the method are investigated in Section 5. Numerical results are reported in Section 6. Some conclusions are made in Section 7.

Throughout the paper, all vectors are column vectors, and $^{\mathrm{T}}$ denotes transpose. $\mathcal{R}^n$ denotes the space of n-dimensional real column vectors, and $\mathcal{R}^n_+$ (respectively, $\mathcal{R}^n_{++}$) denotes the non-negative (respectively, positive) orthant in $\mathcal{R}^n$. For convenience, we write $(u^{\mathrm{T}}, v^{\mathrm{T}})^{\mathrm{T}}$ as (u, v) for any vectors $u, v \in \mathcal{R}^n$. I represents the identity matrix with suitable dimension. $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product. int$\mathcal{K}$ denotes the interior of $\mathcal{K}$. For any $x, y \in \mathcal{R}^n$, we write $x \succeq_{\mathcal{K}} y$ (respectively, $x \succ_{\mathcal{K}} y$) if $x - y \in \mathcal{K}$ (respectively, $x - y \in \mathrm{int}\mathcal{K}$). For any square matrices $A, B \in \mathcal{R}^{n \times n}$, we write $A \succeq B$ (respectively, $A \succ B$) if the symmetric part of $A - B$ is positive semidefinite (respectively, positive definite). For any continuously differentiable function $h = (h_1, h_2, \ldots, h_n)^{\mathrm{T}} : \mathcal{R}^m \to \mathcal{R}^m$, we denote its Jacobian by $h'(x) = (\nabla h_1(x), \nabla h_2(x), \ldots, \nabla h_n(x))^{\mathrm{T}}$, where $\nabla h_i(x)$ denotes the gradient of h_i at x for $i = 1, 2, \ldots, m$, and denote the transpose Jacobian of h at x by $\nabla h(x)$. For any $\alpha, \beta > 0, \ \alpha = O(\beta)$ (respectively, $\alpha = o(\beta)$) means that α/β is uniformly bounded (respectively, tends to zero) as $\beta \to 0$.

2. Preliminaries

In this section, we recall some background materials and preliminary results that will be used in the subsequent sections. We start with the Euclidean Jordan algebra associated with the SOC $\mathcal{K}$ which is a basic tool used in this paper.

For any vectors $x = (x_1, \bar{x}), \ s = (s_1, \bar{s}) \in \mathcal{R} \times \mathcal{R}^{n-1}$, their Jordan product associated with the SOC $\mathcal{K}$ is defined by

$$x \circ s := (x^{\mathrm{T}} s, x_1 \bar{s} + s_1 \bar{x}).$$

One easily checks that this operator is commutative and $(\mathcal{R}^n, \circ)$ is an Euclidean Jordan algebra. The identity element under this product is $\mathbf{e} := (1, 0, \ldots, 0)^{\mathrm{T}} \in \mathcal{R}^n$. Given an element $x = (x_1, \bar{x}) \in \mathcal{R} \times \mathcal{R}^{n-1}$, we define the symmetric matrix