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A note on computing the inverse of a triangular Toeplitz matrix

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ABSTRACT

Using trigonometric polynomial interpolation, a fast and effective numerical algorithm for computing the inverse of a triangular Toeplitz matrix with real numbers has been recently proposed (Lin et al., 2004) [7]. The complexity of the algorithm is two fast Fourier transforms (FFTs) and one fast cosine transform (DCT) of $2n$-vectors. In this paper, we present an algorithm with two fast Fourier transforms (FFTs) of $2n$-vectors for calculating the inverse of a triangular Toeplitz matrix with real and/or complex numbers. A theoretical accuracy and error analysis is also considered. Numerical examples are given to illustrate the effectiveness of our method.

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1. Introduction

Let lT_n be an n-by-n lower triangular Toeplitz matrix:

$$lT_n = \begin{pmatrix} t_0 & & & \\ t_1 & t_0 & & \\ \vdots & \ddots & \ddots & \\ t_{n-1} & \cdots & t_1 & t_0 \end{pmatrix}, \tag{1}$$

where $(t_j)_{j=1,\dots,n-1}$ are real and/or complex numbers. Problems related to compute the inverse of a nonsingular lower triangular Toeplitz matrix often appear in several fundamental problems of scientific computing, signal and image processing, etc. [4,8]. To compute the inverse of a lower triangular Toeplitz matrix with real numbers, Lin et al. [7] give an efficient numerical algorithm whose computational complexity is two fast Fourier transforms (FFTs) and one fast cosine transform (DCT) of $2n$-vectors. Other well-known algorithms, i.e. Bini's and Pan–Chen's algorithms for computing the inverse of a lower triangular Toeplitz matrix are given [3,9].

In this paper, a new algorithm is developed for computing the inverse of a triangular Toeplitz matrix with real and/or complex numbers. The key issue of our method is to adopt the framework of approximate matrix inversion and employ techniques based on interpolation via trigonometric polynomials, following an idea proposed by Lin et al. [7]. The complexity of our method for computing the inverse of a triangular Toeplitz matrix with real and/or complex numbers is two FFTs of $2n$-vectors. A theoretical accuracy and error analysis is also considered. Several numerical examples are given to illustrate the effectiveness and stability of the proposed algorithm with respect to the ones provided by the known algorithms.

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The rest of this paper is organized as follows: In the next section, we give some classical results. In Section 3, our algorithm is presented to compute the inverse of a triangular Toeplitz matrix with real and/or complex numbers. A theoretical error analysis is provided in Section 4. In Section 5, some numerical examples are introduced to show the performance of our algorithm. Finally, we make some concluding remarks in Section 6.

2. Some classical results

To make the paper self-contained we provide the following resume of Toeplitz matrices.

Definition 2.1. $T_n = [t_{ij}]_{i,j=0}^{n-1}$ is a Toeplitz matrix if $t_{ij} = t_{i+k,j+k}$ for all positive k (finite), that is, if all the entries of T_n are invariant in their shifts in the diagonal direction, so that the matrix T_n is completely defined by its first row and its first column.

Toeplitz matrix of size n is completely specified by $2n-1$ parameters, thus requiring less storage space than ordinary dense matrices. Moreover, many computations with Toeplitz matrices can be performed faster; this is the case, for instance, for the sum and the product by a scalar. Less trivial examples are given by the following results:

Proposition 2.1 [1]. *The multiplication of a Toeplitz matrix of size n by a vector can be reduced to multiplication of two polynomials of degree at most 2n and performed with a computational cost of* $O(n\log n)$.

Definition 2.2. $Z_f(\mathbf{c}) = Z_{f,m,n}(\mathbf{c}) = [z_{i,j}]$, for a vector $\mathbf{c} = [c_0, \ldots, c_{m-1}]^T$ and for a scalar $f \neq 0$, is an f-circulant $n \times n$ matrix if $z_{i,j} = c_{i-j}$ for $i \geqslant j$; $z_{i,j} = fc_{n+i-j}$ for $i < j$.

Definition 2.3. $lT_n = lT_n(t) = Z_0(t)$ denotes the lower triangular Toeplitz matrix with the first column t, that is, $lT_n = \sum_{i=0}^{n-1} t_i Z^i$ where $t = (t_0, \ldots, t_{n-1})^T$ and Z is the down-shift matrix filled with zeros, except for its first subdiagonal filled with ones.

Lemma 2.1. *The products and inverses of f-circulant* $n \times n$ *matrices are f-circulant* $n \times n$ *matrices.*

Lemma 2.2. *For a lower triangular Toeplitz matrix* lT_n, *we define the polynomial:*

$$p_n(z) = \sum_{k=0}^{n-1} t_i z^k = t_0 + t_1 z + \ldots + t_{n-1} z^{n-1}. \tag{2}$$

Let the Maclaurin series of $p_n^{-1}(z)$ *be given by*

$$p_n^{-1}(z) = \sum_{k=0}^{\infty} v_k z^k, \tag{3}$$

then

$$lT_n^{-1} = \begin{pmatrix} v_0 & & & \\ v_1 & v_0 & & \\ \vdots & \ddots & \ddots & \\ vn-1 & \cdots & v_1 & v_0. \end{pmatrix} \tag{4}$$

Thus in order to obtain lT_n^{-1}, *we only need to compute the coefficients* v_k *for* $k = 0, 1, \ldots, n-1$.

Lemma 2.3. *Replacing z in (2) and (3) by* ρz *we get*

$$p_{n,\rho}(z) = p_n(\rho z) = \sum_{k=0}^{n-1} (t_k \rho^k) z^k \quad \text{and} \quad p_{n,\rho}^{-1}(z) = p_n^{-1}(\rho z) = \sum_{k=0}^{\infty} (v_k \rho^k) z^k.$$

Equivalently, we have

$$\begin{pmatrix} t_0 & & & \\ \rho t_1 & t_0 & & \\ \vdots & \ddots & \ddots & \\ \rho^{n-1} tn-1 & \cdots & \rho t_1 & t_0 \end{pmatrix}^{-1} = \begin{pmatrix} v_0 & & & \\ \rho v_1 & v_0 & & \\ \vdots & \ddots & \ddots & \\ \rho^{n-1} vn-1 & \cdots & \rho v_1 & v_0 \end{pmatrix}.$$

We note that we can choose $\rho \in (0,1)$ *such that* $\sum_{k=0}^{\infty} |v_k \rho^k| < \infty$.

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