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Asymptotically periodic solutions of fractional differential equations

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ABSTRACT

We study the existence of pseudo *S*-asymptotically ω-periodic mild solutions for a class of abstract fractional differential equation.

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1. Introduction

Real systems usually exhibit internal variations or are submitted to external perturbations. In many situation we can assume that these variations are approximately periodic in a broad sense. In the literature have studied several concepts to represent the idea of approximately periodic function. Most of works deal with asymptotically periodic functions and almost periodic functions. In addition it has recently emerged the notion of *S*-asymptotically ω-periodic functions which has been shown to have interesting applications in several branches of differential equations. This has motivated considerable interest in the topic. Such concept was introduced in the literature by Henríquez et al. [33]. In this paper the authors develop a theory of these type of functions in the Banach space setting. In particular, the authors establish the relationship between *S*-asymptotically ω-periodic functions and the class of asymptotically ω-periodic functions. In [34] the authors discuss the existence of *S*-asymptotically ω-periodic mild solutions for some class of abstract neutral functional differential equations with infinite delay. In [22] the authors have studied the existence and uniqueness of *S*-asymptotically ω-periodic and asymptotically ω-periodic solutions to a first-order differential equation with linear part dominated by a Hille–Yosida operator with non dense domain. Moreover, applications to partial differential equations, fractional integro-differential and neutral differential equations are shown. In [19] the authors have studied the existence of *S*-asymptotically ω-periodic mild solutions for certain class of semilinear Volterra equations. In that paper, the authors extend some results for semilinear fractional integro-differential equations considered in [17], and for the semilinear Cauchy problems of first order given in [33]. Furthermore, the authors give some applications to integral equations arising in viscoelasticity theory. In [7] the authors have studied for the first time the existence and uniqueness of an *S*-asymptotically ω-periodic solution for an abstract third order semi-linear differential equation. These results have significance in the study of vibrations of flexible structures possessing internal material damping. To achieve the results they used a mixed method, combining properties of certain strongly continuous families and fixed point theory. In [2] the authors have studied the existence and uniqueness

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of a discrete S-asymptotically ω-periodic solution for functional difference equations. For further literature concerning S-asymptotically ω-periodic functions we refer the reader to [5,9–12,18,20,24,26,35,44]. In addition, in [47] a new space of S-asymptotically ω-periodic functions was introduced. It is called the space of weighted S-asymptotically ω-periodic (or Sv-asymptotically ω-periodic) functions.[3] In that work, the author establishes the conditions under which a Sv-asymptotically ω-periodic function is asymptotically ω-periodic, and also discusses the existence of Sv-asymptotically ω-periodic solutions for an integral abstract Cauchy problem. The author has applied this theory to partial integro-differential equations. In [21] the authors have studied the existence of weighted S-asymptotically ω-periodic mild solutions for a class of abstract fractional integro-differential equation of the form

$$v'(t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} A v(s)ds + f(t, v(t)), \quad t \geqslant 0, \tag{1.1}$$

$$v(0) = u_0 \in X. \tag{1.2}$$

where $1 < \alpha < 2$, $A : D(A) \subseteq X \to X$ is a linear densely defined operator of sectorial type on a complex Banach space X and $f : [0, \infty) \times X \to X$ is an appropriate function.[4] In a paper to appear Pierri and Rolnik [48] have introduced the concept of pseudo S-asymptotically ω-periodic function (see Definition 2.2), and they have studied qualitative properties of this type of functions. In addition they discuss the existence of pseudo S-asymptotically ω-periodic mild solutions for abstract neutral functional equations. Some applications involving ordinary and partial differential equations with delay are presented.

Fractional calculus is the field of mathematical analysis which deals with the investigation and applications of integrals and derivatives of arbitrary order. We remark that there is much interest in developing theoretical analysis and numerical methods for fractional differential equations because they have recently proved to be valuable in various fields of science and engineering. The strength of derivatives of non-integer order is their ability to describe real situations more adequately than integer order derivatives, especially when the problem has memory or hereditary properties (see [29,49]). For details, including some applications and recent results, see the monographs of Ahn and MacVinish [8], Gorenflo and Mainardi [30], Hilfer [36] and Trujillo et al. [39,45,50], and the recent papers [1,3,4,6,21,27,31,38,40–43,46,52,54].

Abstract fractional differential equation of the form (1.1) and (1.2) are attracting increasing interest. In particular, some properties of the solutions of (1.1) and (1.2) have been studied in several contexts, e. g. well posedness [13], asymptotic behavior [13], asymptotic periodicity [4,7,17,16,25], positivity and contractivity [15]. Nowadays, finding accurate and efficient methods for solving fractional differential equations is an active research topic. It is well known that the exact solutions of most of fractional equations cannot be found easily, thus numerical methods must be used [14,28]. On the other hand the existence of pseudo S-asymptotically ω-periodic (mild) solutions for integro-differential equation of fractional order of type (1.1) remains an untreated topic in the literature. In this work we study sufficient conditions for the existence and uniqueness of a pseudo S-asymptotically ω-periodic (mild) solutions to problem (1.1) and (1.2). This is a significant contribution from the perspective of developing the theory of asymptotic periodicity for fractional evolution equations.

We will now present a summary of this work. We have tried to make the presentation almost self-contained. Section 2 provides the definitions and preliminary results to be used in the theorems stated and proved in the subsequent sections. In particular to facilitate access to the individual topics, in Section 2.1 we review some of the standard properties of sectorial linear operators and the solution operator for fractional equations. In Section 2.2 we present the new space of S-asymptotically ω-periodic functions introduced in [48], which is called the space of pseudo S-asymptotically ω-periodic functions. It is well known that the study of composition of two functions with special properties is very important for deep investigations. We have established a composition result for this type of functions (see Lemma 2.3), which is of central importance in Section 3. Using the formalism presented in Section 2 we deal in Section 3 with very general results on the existence of pseudo S-asymptotically ω-periodic (mild) solutions to the fractional problem (1.1) and (1.2). Finally, in Section 4, we apply our theory to study some concrete applications.

2. Preliminaries and basic results

In this section, we introduce notations, definitions and preliminary facts which are used throughout this work. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be Banach spaces. The notation $\mathcal{B}(X, Y)$ stands for the space of bounded linear operators from X into Y endowed with the uniform operator norm denoted $\|\cdot\|_{\mathcal{B}(X,Y)}$, and we abbreviate the notation to $\mathcal{B}(X)$ and $\|\cdot\|_{\mathcal{B}(X)}$ whenever $X = Y$. In this work $C_b([0,\infty), X)$ denotes the Banach space consisting of all continuous and bounded functions from $[0,\infty)$ into X with the norm of the uniform convergence. For a closed linear operator B, we denote by $\rho(B)$ the resolvent set and by $\sigma(B)$ the spectrum of B (that is, the complement of $\rho(B)$ in the complex plane). In the rest of this work $\omega > 0$ is a fixed real number. We set $B_R(X)$ for the closed ball with center at 0 and radius R in the space X.

[3] Let $v \in C_b([0,\infty),(0,\infty))$. A function $f \in C_b([0,\infty), Z)$ is said to be weighted S-asymptotically ω-periodic if $\lim_{t\to\infty} \frac{f(t+\omega)-f(t)}{v(t)} = 0$.

[4] Note that the convolution integral in (1.1) is known as the Riemann–Liouville fractional integral [13]. By a fractional evolution process we mean a phenomenon governed by an integro-differential equation containing integrals and/ or derivatives of fractional order in time.

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