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Properties and numerical simulations of positive solutions for a variable-territory model ☆

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ABSTRACT

In this paper, we consider a variable-territory predator–prey model with Dirichlet boundary condition. We establish a necessary and sufficient condition for the existence of positive solutions to steady state. Furthermore, we also prove that the local bifurcation positive solutions are unconditional stable. By regular perturbation theorem, we investigate the convergence and stability of positive solutions when handling time m is large enough. At the end of this paper, there are some numerical simulations and biological significance to check and complement our theoretical analysis results.

1. Introduction

In [1], the authors give us a variable-territory predator–prey model such as

$$\begin{aligned} u_t &= u\left(\lambda - au - \frac{bv}{1+mu}\right),\\ v_t &= v\left(\mu - \frac{dv}{u} + \frac{cu}{1+mu}\right). \end{aligned} \tag{1.1}$$

In fact, there is considerable biological evidence that space can affect the dynamics of populations and the structure of communities, thus the corresponding reaction–diffusion equations of (1.1) with the homogeneous Dirichlet boundary condition can be written as

$$\begin{aligned} &u_t - d_1\Delta u = u\left(\lambda - au - \frac{bv}{1+mu}\right), \quad (x,t)\in\Omega\times(0,\infty),\\ &v_t - d_2\Delta v = v\left(\mu - \frac{dv}{u} + \frac{cu}{1+mu}\right), \quad (x,t)\in\Omega\times(0,\infty),\\ &u = v = 0, \quad (x,t)\in\partial\Omega\times(0,\infty),\\ &u(x,0) = u_0(x) \geqslant 0,\ \not\equiv 0,\ v(x,0) = v_0(x) \geqslant 0,\ \not\equiv 0, \quad x\in\bar{\Omega}, \end{aligned} \tag{1.2}$$

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where $u(x,t)$ and $v(x,t)$ respectively represent the species densities of the prey and predator. Ω is a bounded region with smooth boundary in R^n. $\lambda > 0$ and $\mu \in R$, all the other parameters appearing in model (1.2) are assumed to be nonnegative constants. The constants $d_i (i = 1, 2)$ are the diffusion coefficients corresponding to u and v, respectively. λ and μ are their intrinsic/death growth rates. The term $f(s) = \frac{s}{1+ms}$ is the Holling II functional response, where $m > 0$ is the handling time for a generic predator to kill and consume a generic prey. $\frac{d}{u}$ is assumed that self-limitation of the predator depends inversely on the availability of prey. The initial datas $u_0(x)$ and $v_0(x)$ are continuous functions. For the more detailed biological backgrounds of the model, one can refer to [1–5] and the references therein.

In our work, we set $d_1 = d_2 = 1$ and $\Omega = (0, l)$, where $l > 0$. By introducing the following non-dimensional variables

$$\bar{u} = au,\ \bar{v} = adv,\ \bar{m} = m/a,\ \bar{c} = c/a,\ \bar{b} = b/ad,\ \bar{a} = \lambda,\ \bar{d} = \mu,$$

and dropping all superscripts for simplicity, system (1.2) becomes

$$\begin{aligned} &u_t - u_{xx} = u\left(a - u - \frac{bv}{1+mu}\right), \quad (x,t) \in (0,l) \times (0,\infty),\\ &v_t - v_{xx} = v\left(d - \frac{v}{u} + \frac{cu}{1+mu}\right), \quad (x,t) \in (0,l) \times (0,\infty),\\ &u(0,t) = u(l,t) = v(0,t) = v(l,t) = 0, \quad t \in (0,\infty),\\ &u(x,0) = u_0(x) \geqslant 0,\ v(x,0) = v_0(x) \geqslant 0,\ \not\equiv 0, \quad x \in [0,l]. \end{aligned} \tag{1.3}$$

We remark that $a > 0$ and $d \in R$. d is growth rate if $d \geqslant 0$ and death rate if $d < 0$. The variable-territory model has been studied by many mathematicians such as Du and Shi [2], Wang and Pang [3], and Pang and Zhou [6,7]. For $d = -1$, $c = 1$ and $m = 0$, the authors Wang, Pang study the asymptotic behavior and properties of positive solutions of (1.3) with homogeneous Neumann boundary condition on $n (n \geqslant 1)$ dimensional case. We also point out that (1.3) has not been studied with homogeneous Dirichlet boundary condition until now.

The steady state system of (1.3) is given by the following elliptic system

$$\begin{aligned} &-u'' = u\left(a - u - \frac{bv}{1+mu}\right), \quad x \in (0,l),\\ &-v'' = v\left(d - \frac{v}{u} + \frac{cu}{1+mu}\right), \quad x \in (0,l),\\ &u(0) = u(l) = v(0) = v(l) = 0. \end{aligned} \tag{1.4}$$

The goal of this paper is to study system (1.4), the rest of this paper is organized as follows. In Section 2, we give preliminary materials on eigenvalue problem, priori estimate and a necessary condition for the existence of positive solutions of (1.4). In Section 3, we discuss the existence of positive solution by bifurcation theory. In Section 4, we will prove that the positive solution is unconditional locally stable. In Section 5, some properties of positive solutions to (1.4) are studied as handling time m is large. In Section 6, we give some examples to check and complement our theoretical analysis results by numerical simulation.

2. Preliminaries

It is well known that the following problem (see [8], pp1341)

$$-\phi'' = \lambda\phi, \quad x \in (0,l);\ \phi(0) = \phi(l) = 0 \tag{2.1}$$

has an infinite sequence of eigenvalues $\{\lambda_n\}$ such that $0 < \lambda_1 < \lambda_2 \leqslant \lambda_3 \leqslant \cdots$ with corresponding eigenfunctions $\phi_1, \phi_2, \phi_3, \ldots$, where $\phi_1 > 0$ for $x \in (0,l)$. Suppose that $q : [0,l] \to R$ is a $C[0,l]$ function. Then the linear eigenvalue problem

$$-\mu\phi'' + q\phi = \lambda\phi, \quad x \in (0,l); \quad \phi(0) = \phi(l) = 0, \quad \mu \in R, \quad \mu \geqslant \varepsilon > 0 \tag{2.2}$$

also has an infinite sequence of eigenvalues which are bounded below. In the following sections we denote the ith eigenvalue of (2.2) by $\lambda_i(\mu, q)$. It is known that $\lambda_1(\mu, q)$ is a simple eigenvalue and that the corresponding eigenfunctions do not change sign on $(0,l)$. By the variational principle we have

$$\lambda_1(\mu, q) = \inf_{\phi \in W_0^{1,2}} \frac{\mu \int_0^l |\phi'|^2 dx + \int_0^l q\phi^2 dx}{\int_0^l \phi^2 dx}$$

and the following statements as

(Q_1) $\lambda_1(\mu_1, q) > \lambda_1(\mu_2, q)$ for $\mu_1 > \mu_2$.
(Q_2) $\lambda_1(\mu, q_1) > \lambda_1(\mu, q_2)$ for $q_1(x) \geqslant q_2(x)$ and $q_1(x) \not\equiv q_2(x)$.
(Q_3) $\lambda_1(\mu, q + m) = \lambda_1(\mu, q) + m$, where $m \in R$.