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A super accurate shifted Tau method for numerical computation of the Sobolev-type differential equation with nonlocal boundary conditions

B. Soltanalizadeh [a,*], H. Roohani Ghehsareh [c], S. Abbasbandy [b]

[a] Department of Mathematics, University of Houston, 4800 Calhoun Rd, Houston, TX 77204, USA
[b] Department of Mathematics, Imam Khomeini International University, Ghazvin 34149, Iran
[c] Department of Mathematics, Malek Ashtar University of Technology, Shahin Shahr, Isfahan, 83145-115, Iran

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ABSTRACT

In this article, we propose a super accurate numerical scheme to solve the one-dimensional Sobolev type partial differential equation with an initial and two nonlocal integral boundary conditions. Our proposed methods are based on the shifted Standard and shifted Chebyshev Tau method. Firstly, We convert the model of partial differential equation to a linear algebraic equation and then we solve this system. Shifted Standard and shifted Chebyshev polynomials are applied for giving the computational results. Numerical results are presented for some problems to demonstrate the usefulness and accuracy of this approach. The method is easy to apply and produces very accurate numerical results.

1. Introduction

The first results on differential equations with nonlocal integral condition were obtained by Cannon [1] and Batten [2] independently. These nonlocal conditions arise mainly when the data on the boundary cannot be measured directly. Since certain problems of modern physics and technology can be effectively described in terms of nonlocal problems for partial differential equations, then recently these types of equations have been given considerable attention, and various methods have been developed for the treatment of these equations. Theoretical discussions of partial differential equations with nonlocal boundary conditions have been studied in [3–5]. Authors of [6–11] presented some numerical methods for solving these type of equations. Some matrix formulation techniques are proposed for the numerical computation of Heat, Wave and Telegraph equations with integral boundary conditions [12–14].

In this study, we focus on the following Sobolev-type equation given in [11] as

$$u_t - \alpha u_{xx} - \beta u_{txx} = f(x,t), \quad 0 \leqslant x \leq 1, \quad t > 0, \tag{1}$$

with the initial condition

$$u(x,0) = r(x), \quad 0 \leqslant x \leq 1 \tag{2}$$

and nonlocal boundary conditions

* Corresponding author.
E-mail addresses: babak@math.uh.edu, babak.soltanalizadeh@gmail.com (B. Soltanalizadeh).

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$$u(0,t)=\int_0^1 a(x)u(x,t)dx+p(t),\quad t>0, \tag{3}$$

$$u(1,t)=\int_0^1 b(x)u(x,t)dx+q(t),\quad t>0, \tag{4}$$

where the functions $f(x,t), r(x), a(x), b(x), p(t)$ and $q(t)$ and the constants α and β are known and $u(x,t)$ is an unknown function.

Sobolev-type differential equations appears in a variety of physical problems such as flow of fluid through fissured rocks, thermodynamics and propagation of long waves of small amplitude.

The first discussion on Sobolev equation was provided in [19]. Authors of [20,21], presented the existence of solutions of semilinear evolution equations of Sobolev type in Banach space. In [22], the existence of mild and strong solutions of non-local integro-differential Sobolev type equation was proved by using semigroup theory and Schauder fixed point theorem. Bouziani Abdelfatah and Nabil [23], discussed the theoretical of above type equations by applying the Rothe time discretization method. There are only few papers dealing with the numerical methods to solve Sobolev type partial differential equations with nonlocal boundary conditions. Recently, Dubey presented a numerical approximate solution based on the Laplace transform method for the Eqs. (1)–(4) [11].

The aim of this research is presenting a numerical method for solving Eqs. (1)–(4) by using shifted Standard and shifted Chebyshev Tau method with a very hight accuracy. The concept of the Tau method was first proposed by Ortiz and Samara [26] that they proposed an operational technique for the numerical solution of nonlinear ordinary differential equations with some supplementary conditions based on the Tau Method [27]. Then this technique has been described for the case of linear ordinary differential eigenvalue problems [28], integro-differential equations [29–32], partial differential equations [33,34] and for the iterated solutions of linear operator equations [35]. Similar works can be found in [15–18,24,25,36,37].

2. Reformulation of the problem

In this section, we approximate the functions $f(x,t)$, $r(x), a(x), b(x), p(t)$ and $q(t)$ by using two variate Taylor and Chebyshev series. We suppose that these functions are given smooth real valued functions, then they can be approximated by polynomials to any degree of accuracy. To this end, we have

$$\begin{cases} f(x,t)\simeq \sum_{i=0}^{n}\sum_{j=0}^{m} f_{ij}\upsilon_i(x)\omega_j(t)=\upsilon^T F\omega, \quad r(x)\simeq \sum_{i=0}^{n} r_i\upsilon_i(x)=\upsilon^T R,\\ a(x)\simeq \sum_{i=0}^{n} a_i\upsilon_i(x)=\upsilon^T A, \quad b(x)\simeq \sum_{i=0}^{n} b_i\upsilon_i(x)=\upsilon^T B,\\ p(t)\simeq \sum_{j=0}^{m} p_j\omega_j(t)=P\omega, \quad q(t)\simeq \sum_{j=0}^{m} q_j\omega_j(t)=Q\omega, \end{cases} \tag{5}$$

where

$$\begin{cases} R=[r_0,r_1,r_2,\ldots,r_n]^T, \quad A=[a_0,a_1,a_2,\ldots,a_n]^T, \quad B=[b_0,b_1,b_2,\ldots,b_n]^T,\\ P=[p_0,p_1,p_2,\ldots,p_m], \quad Q=[q_0,q_1,q_2,\ldots,q_m],\\ F=[F_0,F_1,F_2,\ldots,F_m], \quad F_i=[f_{0j},f_{1j},f_{2j},\ldots,f_{nj}]^T, \quad j=0,1,2,\ldots,m. \end{cases} \tag{6}$$

Then, we consider the approximate solution of the above problem in the following form:

$$U_{n,m}(x,t)=\sum_{i=0}^{n}\sum_{j=0}^{m} u_{ij}\upsilon_i(x)\omega_j(t)=\upsilon^T U\omega, \tag{7}$$

where

$$U=[U_0,U_1,U_2,\ldots,U_m] \tag{8}$$

and

$$U_j=[u_{0j},u_{1j},u_{2j},\ldots,u_{nj}]^T.$$

The matrix U is an $(n+1)\times(m+1)$ matrix which contains $(n+1)\times(m+1)$ unknown coefficients. We utilize the following proceed for finding these unknowns.

By using Eqs. (5) and (7) in initial condition, we obtain

$$\upsilon^T U\omega(0)=\upsilon^T R,$$

then we have